2022 Route 81 Conference — Schedule —

Saturday, 24 September 2022

Department of Mathematics and Statistics
Queen's University, Kingston, Ontario

09:00 – 09:30	COFFEE AND SNACKS
09:30 – 10:15	JEREMY USATINE (Brown) Motivic integration for Artin stacks
10:30 – 11:15	PORTIA ANDERSON (Cornell) Schubert Calculus with Puzzles: a dialogue between geometry and combinatorics
11:15 – 11:45	BREAK
11:45 – 12:30	NASRIN ALTAFI (Queen's) Hilbert functions of Artinian Gorenstein algebras with Lefschetz properties
12:30 – 14:00	LUNCH
14:00 – 14:45	LAUREN HELLER (Berkeley) Multigraded regularity and betti numbers
15:00 – 15:45	MICHAEL DEBELLEVUE (Syracuse) Rigidity of Graded Deviations and the Slope of an Algebra
15:45 – 16:15	BREAK
16:15 – 17:00	MIKE STILLMAN (Cornell) Gorenstein Calabi-Yau 3-folds and quartic hypersurfaces

All talks are in 234 Jeffery Hall (it is one floor below ground).

Abstracts

Nasrin Altafi *Hilbert functions of graded Artinian Gorenstein algebras with Lefschetz properties*
A graded Artinian algebra is said to satisfy the strong Lefschetz property (SLP) if multiplication by all powers of a general linear form has maximal rank in every degree and if this property holds for the first power the algebra has the weak Lefschetz property (WLP). Determining the Lefschetz properties for Artinian algebras is motivated by the Hard Lefschetz Theorem. The algebraic analog of the cohomology rings of smooth projective varieties investigated by the Hard Lefschetz Theorem are graded Artinian Gorenstein algebras. In this talk, after providing some background material on the subject, I'll present a complete classification of the Hilbert functions of graded Artinian Gorenstein algebras satisfying the SLP.

Portia Anderson *Schubert Calculus with Puzzles: a dialogue between geometry and combinatorics*
Classically, Schubert calculus is about computing the structure constants in the cohomology ring of the Grassmannian by counting the points in triple intersections of Schubert varieties. Knutson–Tao puzzles are fun combinatorial gadgets that compute the same constants, thus bridging the worlds of combinatorics and algebraic geometry. Symmetries that are readily observed in puzzles can sometimes

illuminate more obscure geometric phenomena, or vice versa. In this talk, we will look at some of this interplay with a few classic examples, as well as a recent result on symmetries of parallelogram-shaped puzzles.

Michael DeBellevue *Rigidity of Graded Deviations and the Slope of an Algebra*

The graded deviations of a graded ring are numerical invariants measuring the growth of the resolution of the residue field of a graded ring. Their values reflect certain ring-theoretic properties. In particular, vanishing of off-diagonal deviations is related to the Koszul property. In the classical ungraded case, deviations are known to be “rigid” in the sense that if they are eventually zero, then they must be immediately zero and the ring must be a complete intersection. We present some preliminary results in support of a similar statement for off-diagonal deviations. Some consequences for other numerical invariants, such as the slope of an algebra, will also be discussed.

Lauren Heller *Multigraded regularity and betti numbers*

Castelnuovo-Mumford regularity of a sheaf on projective space can be described in terms of either the local cohomology or the betti numbers of the corresponding graded module. For the toric generalization of regularity defined by Maclagan and Smith this equivalence no longer holds. I will present results relating regularity to multigraded betti numbers in order to bound the regularity of powers of ideals and to characterize the regularity of sheaves on products of projective spaces in terms of truncations.

Mike Stillman *Gorenstein Calabi-Yau 3-folds and quartic hypersurfaces*

Over the last 30 years, Calabi-Yau 3-folds in algebraic geometry have been central objects in string theory, as they allow the construction of string theories in theoretical physics. It is conjectured that there are only a finite number of families of (smooth) Calabi-Yau 3-folds. There are many families known, but the full landscape of possible Calabi-Yau families is still unknown (including finiteness).

In this talk, we zero in on the special but natural case when the projective embedding is arithmetically Gorenstein. If the codimension is small (≤ 3), one can classify all of these. For codimension 4, it is more delicate, and this is the case we consider in this talk. The surprising thing (at least it was for me!) is that the possible (graded) Betti tables can be determined completely, and there are not very many of them. The techniques we use involve Macaulay inverse systems and apolar ideals, leading to a beautiful link with quartic surfaces in projective 3-space (all of this will be explained in the talk!). We will also keep things explicit and classical, by computing examples in *Macaulay2*.

This represents joint work with several others: Beihui Yuan, Hal Schenck (`arXiv:2011.10871`) and and Grzegorz Kapustka, Michal Kapustka, Kristian Ranestad, Beihui Yuan, and Hal Schenck (`arXiv:2111.05817`)

Jeremy Usatine *Motivic integration for Artin stacks*

A standard method for studying a singular variety is to resolve it by a smooth variety and to then relate invariants of the singular variety to invariants of the smooth one. Motivic integration provides powerful tools for obtaining such a relationship. Motivated by the McKay correspondence, I will describe a context in which interesting varieties admit natural resolutions of singularities by Artin stacks. This suggests a need for versatile tools in studying these “stacky” resolutions of singularities. I will discuss joint work with M. Satriano in which we use motivic integration to provide such tools, and I will also explain how our work leads to a notion of crepantness for stacky resolutions of singularities.