

Queen's Algebraic Geometry — Seminar —

A TROPICAL APPROACH TO RATIONAL CURVES ON GENERAL HYPERSURFACES

TRISTRAM BOGART
Queen's University

Abstract

In the 1980's, Herbert Clemens made a series of conjectures about the dimensions of spaces of rational curves on general complex hypersurfaces in projective space. The most general of these conjectures is that there are only finitely many rational curves of degree d on a general quintic threefold in \mathbb{P}^4 . He proved that a general hypersurface of degree $2n - 1$ in \mathbb{P}^n contains no rational curves.

In ongoing joint work with Ethan Cotterill, we develop a new approach to these questions via tropical geometry. A tropical curve is a graph embedded in \mathbb{R}^n in such a way that at each vertex, the primitive integer edge directions add up to zero. The curve is rational if the graph is a tree. The tropical hypersurface of a polynomial f is a polyhedral complex dual to a certain subdivision of the Newton polytope of f . Since tropicalization preserves inclusion, the tropical analogue of Clemens' theorem would imply the original theorem. Magnus Vigeland recently produced a family of tropical surfaces in \mathbb{R}^3 of degree d that contain no tropical lines when d is at least 4; our goal is to show that these same surfaces contain no tropical rational curves when d is at least 5. Such a proof, combinatorial in flavor, would have the benefits of being constructive and probably characteristic-free. Our current result is that Vigeland's surfaces contain no tropical rational curves that are generic in a certain sense.

Monday, October 6, 2008
4:30pm – 5:30pm
319 Jeffery Hall