Queen's Algebraic Geometry — Seminar —

Seshadri Constants, Diophantine Approximation, & Roth's Theorem for Arbitrary Varieties

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Abstract

If X is a variety of general type defined over a number field k, then the Bombieri-Lang conjecture predicts that the k-rational points of X are not Zariski dense. One way to view the conjecture is that a global condition on the canonical bundle (that it is "generically positive") implies a global condition about rational points. By a well-established principle in geometry we should also look for local influence of positivity on the local accumulation of rational points. To do that we need measures of both these local phenomena.

Let L be an ample line bundle on X, and x an algebraic point of X. By slightly modifying the usual definition of approximation exponent on the affine line we define a new invariant $\alpha_x(L) \in (0, \infty]$ which measures how quickly rational points accumulate around x, from the point of view of L.

The central theme of the talk is the interrelations between $\alpha_x(L)$ and the Seshadri constant $\epsilon_x(L)$ measuring the local positivity of L near x. In particular, the classic approximation theorems on the line — the theorems of Liouville and Roth — generalize as inequalities between $\alpha_x(L)$ and $\epsilon_x(L)$ valid for all projective varieties. This is joint work with David McKinnon.

Monday 1 October 2012
16:30 – 17:30
319 Jeffery Hall