Queen's Algebraic Geometry
— Seminar —

Outer space and the combinatorics of character varieties

CHRIS MANON
George Mason University

Abstract

For a discrete group π and a reductive group G, the character variety $\mathcal{X}(\pi, G)$ is the moduli space of semi-stable representations of π into G. Many moduli spaces appear in the guise of character varieties, for example for a manifold M with fundamental group $\pi_1(M) = \pi$, $\mathcal{X}(\pi, G)$ is the space of flat topological G-bundles on M, and if M is equipped with a Kähler structure, $\mathcal{X}(\pi, G)$ is the space of G-Higgs bundles. We will discuss several related combinatorial and geometric features of the character variety $\mathcal{X}(F_g, SL_2(\mathbb{C}))$, where F_g is the free group on g generators and $G = SL_2(\mathbb{C})$ is the special linear group of 2×2 matrices. For every choice of graph Γ equipped with an isomorphism $\gamma\colon \pi_1(\Gamma) \cong F_g$ we describe a so-called Newton-Okounkov body for $\mathcal{X}(F_g, SL_2(\mathbb{C}))$, this is a convex cone which informs the structure of the coordinate ring $\mathbb{C}[\mathcal{X}(F_g, SL_2(\mathbb{C}))]$. We also discuss a related integrable system in $\mathcal{X}(F_g, SL_2(\mathbb{C}))$, and a polyhedral cone of discrete valuations on $\mathbb{C}[\mathcal{X}(F_g, SL_2(\mathbb{C}))]$ for each choice of graph (Γ, γ). We conclude by describing how these polyhedra knit together to give a map from Culler and Vogtman's "outer space" into the Berkovich analytification of $\mathcal{X}(F_g, SL_2(\mathbb{C}))$, giving a new proof of a result of Morgan and Shalen.

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