Queen's Algebraic Geometry — Seminar —

Embeddings of semisimple groups and cohomology maps

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Abstract

Let G, $\widetilde{G}$ be two semisimple complex algebraic groups and X, $\widetilde{X}$ be their flag varieties. Suppose $\iota\colon G \hookrightarrow \widetilde{G}$ is an embedding, $\widetilde{B}$ is a Borel subgroup of $\widetilde{G}$, and $B = G \cap \widetilde{B}$ is a Borel subgroup of G. This gives an embedding $X \hookrightarrow \widetilde{X}$. An equivariant line bundle on $\widetilde{X}$ is determined by an integral weight $\widetilde{\lambda}$ of $\widetilde{G}$ and denoted $\mathcal{L}_{\widetilde{\lambda}}$. A line bundle $\mathcal{L}_{\widetilde{\lambda}}$ on $\widetilde{X}$ restricts to a line bundle $\mathcal{L}_\lambda$ on X, and we have a map on sheaf cohomology

$$\pi : H(\widetilde{X}, \mathcal{L}_{\widetilde{\lambda}}) \longrightarrow H(X, \mathcal{L}_\lambda)$$

By the Borel-Weil-Bott theorem these two cohomology spaces are irreducible modules over $\widetilde{G}$ and G respectively. We can consider both as G modules via restriction. The map π is equivariant, and the irreducibility of $H(X, \mathcal{L}_\lambda)$ implies, via Schur's lemma, that π is either surjective or zero.

Dimitrov and Roth have recently found a necessary and sufficient condition for π to be nonzero, in the case of the diagonal embedding $X \hookrightarrow X \times X$. This condition, expressed in terms of the root systems of G and $\widetilde{G}$ suggests a relation to Lie algebra cohomology and the theory developed by Kostant for his alternative proof of the Borel-Weil-Bott theorem.

It turns out that an analogue to π restriction map occurs in Lie algebra cohomology. I will discuss the relation between the two cohomology theories and consider some examples of restriction maps in Lie algebra cohomology, namely those coming from diagonal embeddings and root-embeddings.

Monday, March 30, 2009
4:30pm – 5:30pm
319 Jeffery Hall