Queen's Algebraic Geometry
— Seminar —

The Structure of the Resolutions of Length Three

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Abstract

Let us recall that for a given format $(r_n, \ldots, r_1)$ of the free complex

$$0 \to F_n \to F_{n-1} \to \cdots \to F_0$$

over a commutative ring with the rank of the i-th differential d_i equal to r_i (and thus $\operatorname{rank} F_i = r_r + r_{i+1}$), we say that an acyclic complex F_{gen} over a given ring R_{gen} is generic if for every complex G of this format over a Noetherian ring S there exists a homomorphism $f\colon R_{\mathrm{gen}} \to S$ such that $G = F_{\mathrm{gen}} \otimes_{R_{\mathrm{gen}}} S$. For complexes length 2 the existence of the generic acyclic complex was established by Hochster and Huneke in the 1980's. It is a normalization of the ring giving a generic complex (two matrices with composition zero and rank conditions). I prove the following result: Associate to a triple of ranks (r_3, r_2, r_1) a triple $(p, q, r) = (r_3 + 1, r_2 - 1, r_1 + 1)$. Associate to (p, q, r) the graph $T_{p,q,r}$ (three arms of lengths $p - 1$, $q - 1$, $r - 1$ attached to the central vertex). Then there exists a Noetherian generic ring for this format if and only if $T_{p,q,r}$ is a Dynkin graph. In other cases one can construct in a uniform way a non-Noetherian generic ring, which carries an action of the corresponding Kac-Moody Lie algebra.

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319 Jeffery Hall