Queen's Algebraic Geometry
— Seminar —

Apolarity, Waring ranks, and direct sum decomposability of polynomials

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Abstract

Can the determinant of the n-by-n generic matrix $(x_{i,j})$ be written as a sum of polynomials in linearly independent variables? For $n = 2$, the answer is yes. For $n > 2$, it's easy to see there is no such decomposition using the original variables, but it's harder to answer the question when a linear change of coordinates is allowed.

More generally, a direct sum decomposition of a polynomial F is an expression $F = G + H$ where G and H depend on linearly independent variables, possibly after a change of basis. For example, $F = xy$ is not decomposable in the original basis, but $xy = (1/4)(x + y)^2 - (1/4)(x - y)^2$ gives a direct sum decomposition after a linear change of coordinates. General homogeneous forms are indecomposable, but it is not easy to determine whether a given form is decomposable.

We analyze criteria for decomposability via apolarity, uncovering a surprising connection to Waring rank. This is joint work with Weronika Buczyńska, Jarek Buczyński, and Johannes Kleppe.

Monday 13 January 2014
16:30 – 17:30
319 Jeffery Hall