Queen's Algebraic Geometry — Seminar —

Ideal Generators of Projective Monomial Curves

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Abstract

We consider the number of generators of the ideal of projective monomial curves of degree d in $\mathbb{P}^3$. It seems that there is a probability distribution p_i, $2 \leq i \leq \infty$, such that as $d \to \infty$, the fraction of all curves of degree d with i generators approaches p_i. Experimental evidence suggests that $p_2 = 0$, $p_3 = 0.31$ (the Cohen–Macaulay cases), $p_4 = 0.12$, and $p_5 = 0.11$. Furthermore, the average number of generators grows proportional to $\log(d)$, but the median number of generators is 5 if d is sufficiently large.

We cannot rigorously prove these claims, but we offer the following arguments to support them. The ideal generators are described in terms of the Hilbert bases of the semigroups obtained by intersecting several different lattices in $\mathbb{Z}^2$ with various cones. These Hilbert bases consists of lattice points on straight line segments, each segments contributing q_i generators, where the q_i are quotients of a continued fraction. The number of segments contributing ideal generators, averaged over all curves of degree d, as $d \to \infty$, seems to be positive but somewhat less that one. Therefore, the average number of generators should be distributed in the same way as the quotients of continued fractions, as described by Knuth in The Art of Computer Programming, Volume 2.

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