Abelian Subvarieties and the Shimura Construction

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Abstract:

The purpose of this note is to show that the *Shimura construction* follows from a general "dictionary" that translates statements about subvarieties of abelian varieties into statements about ideals of the associated endomorphism algebras.

Theorem. *Let A/K be an abelian variety with endomorphism algebra $\mathbb{E} = \operatorname{End}^0_K(A)$. Then the map $B \mapsto I(B) := \{f \in \mathbb{E} : \operatorname{Im} f \subset B\}$ defines an inclusion-preserving bijection*

$$I_{A/K} : \mathbf{Sub}(A/K) \xrightarrow{\sim} \mathbf{Id}_{\mathbb{E}}$$

between the set of abelian subvarieties of A/K and the set of right ideals of $\mathbb{E} = \operatorname{End}^0_K(A)$.

This theorem can be generalized to arbitrary finitely generated faithful $\mathbb{E}$-modules V (in place of $\mathbb{E}$), and this leads to a generalization of the Shimura construction.