

# Binary Theta Series and Modular Forms with Complex Multiplication

## Abstract:

The main purpose of this paper is to give an *intrinsic interpretation* of the space  $\Theta(D)$  generated by the binary theta series  $\vartheta_f$  attached to the positive binary quadratic forms  $f$  whose discriminant has the form  $D(f) = D/t^2$ , for some integer  $t$ . It turns out that  $\Theta(D) = M_1^{CM}(|D|, \psi_D)$ , the space of modular forms of weight 1 and of level  $|D|$  which have *complex multiplication* (CM) by their Nebentypus character  $\psi_D = (\frac{D}{\cdot})$ ; cf. Theorem 1. As an application, we obtain a structure theorem (Theorem 16) of the space  $M_1^{CM}(|D|, \psi_D)$ .

The proof of this theorem rests on the results of [8] together with a characterization of the newforms  $f$  which have CM by their Nebentypus character in terms of properties of the associated Deligne/Serre *Galois representation*  $\rho_f$ ; cf. Theorem 9.

## Keywords:

Binary quadratic forms, theta series, modular forms, Hecke algebra, newforms, complex multiplication, Galois representations, dihedral groups.

## 2010 AMS Subject Classification:

**11F27** Number Theory: Discontinuous groups and automorphic forms: Theta series; Weil representation; Theta correspondence

**11F80** Number Theory: Discontinuous groups and automorphic forms: Galois representations

**11E45** Number Theory: Forms and Linear Algebraic Groups: Analytic Theory (Epstein zeta function; relation with automorphic forms and functions)