

Invariants of the Surface $\tilde{Z}_{10,1}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	2	36	40	-30	42	-6

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	L_∞	L	$2S_\infty$	$2S$
	360	13	4	6	6	3	3	6	0	8	17	30	0	0

The Singularities of the associated singular surface $Z_{10,1}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 9]	+	1	1	[10, 9], [1, 1]	[181, 380, 200]	[1, 0, 100]
2	[5, 6]	+	1	2	[5, 6], [-1, -1]	[61, -220, 200]	[4, 0, 25]
3	[0, 7]	-	2	3	[10, 7], [-3, -2]	[149, -880, 1300]	[1, 0, 100]
4	[5, 8]	-	2	3	[5, 8], [-2, -3]	[89, -680, 1300]	[4, 0, 25]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -10, 100]	[1, 0, 75]
6	[1, 4]	+	2	2	[1, 4], [0, 1]	[21, 90, 100]	[4, 2, 19]
7	[1, 5]	+	2	2	[1, 5], [0, 1]	[31, 110, 100]	[4, -2, 19]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
8	[0, 3]	-	3	3	[10, 3], [3, 1]	[139, 850, 1300]	[1, 0, 75]
9	[2, 3]	-	3	3	[2, 3], [-1, -1]	[19, -150, 300]	[4, -2, 19]
10	[3, 2]	-	3	3	[3, 2], [1, 1]	[19, 150, 300]	[4, 2, 19]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	[1, 0]	1	1	[10, 1]	1	[10]
12	[1, 2]	2	2	[2, 1]	1	[2]
13	[1, 4]	2	2	[2, 1]	1	[2]
14	[1, 5]	1	3	[5, 1]	1	[5]
15	[2, 5]	1	4	[5, 4]	4	[2, 2, 2, 2]
16	[3, 0]	1	5	[10, 9]	9	[2, 2, 2, 2, 2, 2, 2, 2, 2]

The Basic Curves on $\tilde{Z}_{10,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	6	6	0	-2
6	6	6	0	-2
7	3	3	0	-3
17	3	3	0	-3
18	0	0	0	-3
36	0	0	0	-3

The intersection matrix for the non-exceptional curves:

No	1	6	7	17	18	36
1	-2	88	0	60	0	18
6	88	-2	60	0	18	0
7	0	60	-3	38	0	12
17	60	0	38	-3	12	0
18	0	18	0	12	-3	2
36	18	0	12	0	2	-3

The intersection matrix for the P_0 -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	88
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	88	1	1	1	1	-2

The intersection matrix for the P_1 -curves (curves 7...17)

No	7	8	9	10	11	12	13	14	15	16	17
7	-3	1	1	1	1	0	1	0	1	0	38
8	1	-3	0	0	0	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	0	0	0	1
10	1	0	0	-3	0	0	0	0	0	0	1
11	1	0	0	0	-2	1	0	0	0	0	0
12	0	0	0	0	1	-2	0	0	0	0	1
13	1	0	0	0	0	0	-2	1	0	0	0
14	0	0	0	0	0	0	1	-2	0	0	1
15	1	0	0	0	0	0	0	0	-2	1	0
16	0	0	0	0	0	0	0	0	1	-2	1
17	38	1	1	1	0	1	0	1	0	1	-3

The intersection matrix for the P_∞ -curves (curves 18...36)

No	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
18	-3	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	2
19	1	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
20	1	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	1	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
22	1	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	1	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	1
27	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
36	2	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	-3

**The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{10,1}$ for $n \leq 30$
 Their basic properties:**

No	n	k	deg	p_a	g_T	δ	T^2
37	1	1	1	0	0	0	-1
38	9	3	12	0	0	0	-2
39	11	1	12	1	1	0	0
40	19	3	20	1	1	0	-2
41	21	1	32	1	1	0	-2
42	29	3	30	3	2	1	0

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6
37	1	1	1	0	1	0	0	0	0
38	9	3	12	6	0	0	0	0	6
39	11	1	12	6	0	0	0	0	6
40	19	3	20	10	0	0	0	0	10
41	21	1	32	16	0	0	0	0	16
42	29	3	30	14	0	2	0	0	14

b) Those with the curves over P_1 :

No	n	k	deg	7	8	9	10	11	12	13	14	15	16	17
37	1	1	1	0	1	0	0	0	0	0	0	0	0	0
38	9	3	12	4	0	0	0	0	0	0	0	0	0	4
39	11	1	12	4	0	0	0	0	0	0	0	0	0	4
40	19	3	20	6	0	1	1	0	0	0	0	0	0	6
41	21	1	32	10	0	1	1	0	0	0	0	0	0	10
42	29	3	30	10	0	0	0	0	0	0	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
37	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	9	3	12	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
39	11	1	12	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
40	19	3	20	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1
41	21	1	32	2	2	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	2
42	29	3	30	2	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	2

d) Those of the Hecke curves with each other:

No	n	k	deg	37	38	39	40	41	42
37	1	1	1	-1	0	0	0	0	0
38	9	3	12	0	-2	0	0	0	0
39	11	1	12	0	0	0	2	2	2
40	19	3	20	0	0	2	-2	0	2
41	21	1	32	0	0	2	0	-2	4
42	29	3	30	0	0	2	2	4	0