

Invariants of the Surface $\tilde{Z}_{10,3}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	2	31	35	-25	37	-1

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	L_∞	L	$2S_\infty$	$2S$
	360	13	4	6	6	3	3	6	0	8	12	25	0	0

The Singularities of the associated singular surface $Z_{10,3}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[2, 7]	+	2	1	[2, 7], [-1, -3]	[53, -460, 1000]	[8, 4, 13]
2	[3, 8]	+	2	1	[3, 8], [1, 3]	[73, 540, 1000]	[8, -4, 13]
3	[1, 6]	-	2	2	[1, 6], [0, 1]	[37, 120, 100]	[8, -4, 13]
4	[4, 9]	-	2	2	[4, 9], [-1, -2]	[97, -440, 500]	[8, 4, 13]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	[1, 1]	+	1	1	[1, 1], [-1, 0]	[3, -30, 100]	[3, 0, 25]
6	[1, 3]	+	2	2	[1, 3], [0, 1]	[13, 70, 100]	[7, -6, 12]
7	[3, 1]	+	2	2	[3, 1], [-1, 0]	[13, -70, 100]	[7, 6, 12]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
8	[1, 2]	-	3	3	[1, 2], [0, 1]	[7, 50, 100]	[7, -6, 12]
9	[2, 1]	-	3	3	[2, 1], [-1, 0]	[7, -50, 100]	[7, 6, 12]
10	[3, 3]	-	3	3	[3, 13], [-1, -4]	[217, -1350, 2100]	[3, 0, 25]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	[1, 0]	1	1	[10, 3]	3	[4, 2, 2]
12	[1, 2]	2	2	[2, 1]	1	[2]
13	[1, 4]	2	2	[2, 1]	1	[2]
14	[1, 5]	1	3	[5, 3]	2	[2, 3]
15	[2, 5]	1	4	[5, 2]	2	[3, 2]
16	[3, 0]	1	5	[10, 7]	3	[2, 2, 4]

The Basic Curves on $\tilde{Z}_{10,3}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	6	6	0	-2
6	6	6	0	-2
7	3	3	0	-3
17	3	3	0	-3
18	0	0	0	-3
31	0	0	0	-3

The intersection matrix for the non-exceptional curves:

No	1	6	7	17	18	31
1	-2	88	0	60	0	18
6	88	-2	60	0	18	0
7	0	60	-3	38	0	12
17	60	0	38	-3	12	0
18	0	18	0	12	-3	2
31	18	0	12	0	2	-3

The intersection matrix for the P_0 -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	88
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	88	1	1	1	1	-2

The intersection matrix for the P_1 -curves (curves 7...17)

No	7	8	9	10	11	12	13	14	15	16	17
7	-3	1	1	1	1	0	1	0	1	0	38
8	1	-3	0	0	0	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	0	0	0	1
10	1	0	0	-3	0	0	0	0	0	0	1
11	1	0	0	0	-2	1	0	0	0	0	0
12	0	0	0	0	1	-2	0	0	0	0	1
13	1	0	0	0	0	0	-2	1	0	0	0
14	0	0	0	0	0	0	1	-2	0	0	1
15	1	0	0	0	0	0	0	0	-2	1	0
16	0	0	0	0	0	0	0	0	1	-2	1
17	38	1	1	1	0	1	0	1	0	1	-3

The intersection matrix for the P_∞ -curves (curves 18...31)

No	18	19	20	21	22	23	24	25	26	27	28	29	30	31
18	-3	1	0	0	1	1	1	0	1	0	1	0	0	2
19	1	-4	1	0	0	0	0	0	0	0	0	0	0	0
20	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
21	0	0	1	-2	0	0	0	0	0	0	0	0	0	1
22	1	0	0	0	-2	0	0	0	0	0	0	0	0	1
23	1	0	0	0	0	-2	0	0	0	0	0	0	0	1
24	1	0	0	0	0	0	-2	1	0	0	0	0	0	0
25	0	0	0	0	0	0	1	-3	0	0	0	0	0	1
26	1	0	0	0	0	0	0	0	-3	1	0	0	0	0
27	0	0	0	0	0	0	0	0	1	-2	0	0	0	1
28	1	0	0	0	0	0	0	0	0	0	-2	1	0	0
29	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
30	0	0	0	0	0	0	0	0	0	0	0	1	-4	1
31	2	0	0	1	1	1	0	1	0	1	0	0	1	-3

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{10,3}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
32	3	3	4	0	0	0	-1
33	7	1	8	0	0	0	-2
34	13	3	14	0	0	0	-2
35	17	1	18	1	1	0	-2
36	23	3	24	2	2	0	0
37	27	1	36	2	1	1	0

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6
32	3	3	4	2	0	0	0	0	2
33	7	1	8	4	0	0	0	0	4
34	13	3	14	6	1	1	0	0	6
35	17	1	18	8	1	1	0	0	8
36	23	3	24	12	0	0	0	0	12
37	27	1	36	18	0	0	0	0	18

b) Those with the curves over P_1 :

No	n	k	deg	7	8	9	10	11	12	13	14	15	16	17
32	3	3	4	1	1	0	0	0	0	0	0	0	0	1
33	7	1	8	2	0	1	1	0	0	0	0	0	0	2
34	13	3	14	4	0	1	1	0	0	0	0	0	0	4
35	17	1	18	6	0	0	0	0	0	0	0	0	0	6
36	23	3	24	8	0	0	0	0	0	0	0	0	0	8
37	27	1	36	12	0	0	0	0	0	0	0	0	0	12

c) Those with the curves over P_∞ :

No	n	k	deg	18	19	20	21	22	23	24	25	26	27	28	29	30	31
32	3	3	4	0	1	0	0	0	0	0	0	0	0	0	0	1	0
33	7	1	8	0	0	0	1	0	0	0	0	0	0	1	0	0	0
34	13	3	14	1	1	0	0	0	0	0	0	0	0	0	0	1	1
35	17	1	18	1	0	0	1	0	0	0	0	0	0	1	0	0	1
36	23	3	24	2	1	0	0	0	0	0	0	0	0	0	0	1	2
37	27	1	36	2	2	0	1	0	0	0	0	0	0	1	0	2	2

d) Those of the Hecke curves with each other:

No	n	k	deg	32	33	34	35	36	37
32	3	3	4	-1	0	0	0	0	0
33	7	1	8	0	-2	0	0	0	0
34	13	3	14	0	0	-2	0	2	0
35	17	1	18	0	0	0	-2	4	2
36	23	3	24	0	0	2	4	0	4
37	27	1	36	0	0	0	2	4	0