

Invariants of the Surface $\tilde{Z}_{11,1}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	2	35	39	-29	41	-5

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2S_\infty$	$2S$
	660	26	6	12	4	8	2	5	1	10	15	27	1	1

The Singularities of the associated singular surface $Z_{11,1}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 10]	+	1	1	[11, 10], [1, 1]	[221, 462, 242]	[1, 0, 121]
2	[3, 6]	+	2	2	[3, 17], [1, 6]	[298, 2310, 4477]	[5, 4, 25]
3	[5, 8]	+	2	2	[5, 8], [-2, -3]	[89, -748, 1573]	[5, -4, 25]
4	[1, 8]	-	3	3	[1, 8], [0, 1]	[65, 176, 121]	[10, -6, 13]
5	[3, 10]	-	3	3	[3, 10], [-1, -3]	[109, -726, 1210]	[10, 6, 13]
6	[4, 7]	-	3	3	[4, 7], [1, 2]	[65, 396, 605]	[2, 2, 61]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
7	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -11, 121]	[1, 1, 91]
8	[2, 2]	+	1	2	[2, 13], [-1, -6]	[199, -2035, 5203]	[3, 3, 31]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[1, 4]	-	2	3	[1, 4], [0, 1]	[21, 99, 121]	[7, -1, 13]
10	[1, 6]	-	2	3	[1, 6], [0, 1]	[43, 143, 121]	[7, 1, 13]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	[1, 0]	1	1	[11, 1]	1	[11]
12	[2, 0]	1	2	[11, 3]	2	[4, 3]
13	[3, 0]	1	3	[11, 5]	5	[3, 2, 2, 2, 2]
14	[4, 0]	1	4	[11, 9]	5	[2, 2, 2, 2, 3]
15	[5, 0]	1	5	[11, 4]	2	[3, 4]

The Basic Curves on $\tilde{Z}_{11,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	12	12	0	-3
8	12	12	0	-3
9	8	8	0	-2
16	8	8	0	-2
17	1	1	0	-2
33	1	1	0	-2

The intersection matrix for the non-exceptional curves:

No	1	8	9	16	17	33
1	-3	162	0	110	0	30
8	162	-3	110	0	30	0
9	0	110	-2	72	0	20
16	110	0	72	-2	20	0
17	0	30	0	20	-2	5
33	30	0	20	0	5	-2

The intersection matrix for the P_0 -curves (curves 1...8)

No	1	2	3	4	5	6	7	8
1	-3	1	1	1	1	1	1	162
2	1	-2	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	1
4	1	0	0	-2	0	0	0	1
5	1	0	0	0	-2	0	0	1
6	1	0	0	0	0	-2	0	1
7	1	0	0	0	0	0	-2	1
8	162	1	1	1	1	1	1	-3

The intersection matrix for the P_1 -curves (curves 9...16)

No	9	10	11	12	13	14	15	16
9	-2	1	1	1	0	1	0	72
10	1	-3	0	0	0	0	0	1
11	1	0	-3	0	0	0	0	1
12	1	0	0	-2	1	0	0	0
13	0	0	0	1	-2	0	0	1
14	1	0	0	0	0	-2	1	0
15	0	0	0	0	0	1	-2	1
16	72	1	1	0	1	0	1	-2

The intersection matrix for the P_∞ -curves (curves 17...33)

No	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
17	-2	1	1	0	1	0	0	0	0	1	0	0	0	0	1	0	5
18	1	-11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
19	1	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	1
21	1	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	1
26	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0	1
31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1
33	5	1	0	1	0	0	0	0	1	0	0	0	0	1	0	1	-2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{11,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
34	1	1	1	0	0	0	-1
35	3	2	4	0	0	0	-1
36	4	5	6	0	0	0	-1
37	5	3	6	0	0	0	-2
38	9	4	12	0	0	0	-2
39	12	1	24	0	0	0	-2
40	14	2	24	1	1	0	-2
41	15	5	24	1	1	0	-2
42	16	3	24	0	0	0	-2
43	20	4	36	1	1	0	-2
44	23	1	24	2	2	0	0
45	25	2	30	0	0	0	-2
46	26	5	42	2	2	0	0
47	27	3	36	1	1	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8
34	1	1	1	0	1	0	0	0	0	0	0
35	3	2	4	2	0	0	0	0	0	0	2
36	4	5	6	3	0	0	0	0	0	0	3
37	5	3	6	2	0	1	1	0	0	0	2
38	9	4	12	6	0	0	0	0	0	0	6
39	12	1	24	12	0	0	0	0	0	0	12
40	14	2	24	12	0	0	0	0	0	0	12
41	15	5	24	12	0	0	0	0	0	0	12
42	16	3	24	12	0	0	0	0	0	0	12
43	20	4	36	18	0	0	0	0	0	0	18
44	23	1	24	12	0	0	0	0	0	0	12
45	25	2	30	14	0	1	1	0	0	0	14
46	26	5	42	20	0	1	1	0	0	0	20
47	27	3	36	18	0	0	0	0	0	0	18

b) Those with the curves over P_1 :

No	n	k	deg	9	10	11	12	13	14	15	16
34	1	1	1	0	1	0	0	0	0	0	0
35	3	2	4	1	0	1	0	0	0	0	1
36	4	5	6	2	0	0	0	0	0	0	2
37	5	3	6	2	0	0	0	0	0	0	2
38	9	4	12	4	0	0	0	0	0	0	4
39	12	1	24	8	0	0	0	0	0	0	8
40	14	2	24	8	0	0	0	0	0	0	8
41	15	5	24	8	0	0	0	0	0	0	8
42	16	3	24	8	0	0	0	0	0	0	8
43	20	4	36	12	0	0	0	0	0	0	12
44	23	1	24	8	0	0	0	0	0	0	8
45	25	2	30	10	0	0	0	0	0	0	10
46	26	5	42	14	0	0	0	0	0	0	14
47	27	3	36	12	0	0	0	0	0	0	12

c) Those with the curves over P_∞ :

No	n	k	deg	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
34	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	3	2	4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
36	4	5	6	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
37	5	3	6	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
38	9	4	12	0	2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
39	12	1	24	1	2	1	0	0	1	0	0	0	0	0	0	1	0	0	1	1
40	14	2	24	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1
41	15	5	24	1	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	1
42	16	3	24	1	2	0	1	1	0	0	0	0	0	0	0	0	1	1	0	1
43	20	4	36	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1
44	23	1	24	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
45	25	2	30	2	4	1	0	0	0	0	0	0	0	0	0	0	0	0	1	2
46	26	5	42	3	4	0	1	0	0	0	0	0	0	0	0	0	0	1	0	3
47	27	3	36	2	0	2	0	1	0	0	0	0	0	0	0	0	1	0	2	2

d) Those of the Hecke curves with each other:

No	n	k	deg	34	35	36	37	38	39	40	41	42	43	44	45	46	47
34	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
35	3	2	4	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
36	4	5	6	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
37	5	3	6	0	0	0	-2	0	0	0	0	0	0	0	0	0	0
38	9	4	12	0	0	0	0	-2	0	0	0	0	0	0	0	0	2
39	12	1	24	0	0	0	0	0	-2	0	0	0	0	2	0	2	0
40	14	2	24	0	0	0	0	0	0	-2	0	2	0	2	2	4	0
41	15	5	24	0	0	0	0	0	0	0	-2	0	0	2	2	2	2
42	16	3	24	0	0	0	0	0	0	2	0	-2	0	2	0	0	2
43	20	4	36	0	0	0	0	0	0	0	0	0	-2	4	2	4	0
44	23	1	24	0	0	0	0	0	2	2	2	2	4	0	2	2	4
45	25	2	30	0	0	0	0	0	0	2	2	0	2	2	-2	0	2
46	26	5	42	0	0	0	0	0	2	4	2	0	4	2	0	0	6
47	27	3	36	0	0	0	0	2	0	0	2	2	0	4	2	6	-2