

# Invariants of the Surface $\tilde{Z}_{11,2}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	3	40	46	-32	48	0

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$L_\infty$	$L$	$2S_\infty$	$2S$
	660	26	6	12	4	8	2	5	1	10	22	34	-1	-1

## The Singularities of the associated singular surface $Z_{11,2}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[1, 10]	+	1	1	[1, 10], [0, 1]	[101, 220, 121]	[2, 2, 61]
2	[2, 8]	+	2	2	[2, 19], [-1, -9]	[365, -3806, 9922]	[10, -6, 13]
3	[3, 9]	+	2	2	[3, 20], [1, 7]	[409, 3146, 6050]	[10, 6, 13]
4	[0, 8]	-	3	3	[11, 8], [4, 3]	[185, 1496, 3025]	[1, 0, 121]
5	[2, 7]	-	3	3	[2, 7], [-1, -3]	[53, -506, 1210]	[5, -4, 25]
6	[4, 9]	-	3	3	[4, 9], [-1, -2]	[97, -484, 605]	[5, 4, 25]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
7	[1, 3]	+	2	1	[1, 3], [0, 1]	[13, 77, 121]	[7, 1, 13]
8	[3, 1]	+	2	1	[3, 1], [-1, 0]	[13, -77, 121]	[7, -1, 13]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
9	[0, 3]	-	2	2	[11, 3], [-4, -1]	[163, -1287, 2541]	[1, 1, 91]
10	[1, 5]	-	2	2	[1, 5], [0, 1]	[31, 121, 121]	[3, 3, 31]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	[1, 0]	1	1	[11, 2]	2	[6, 2]
12	[2, 0]	1	2	[11, 6]	2	[2, 6]
13	[3, 0]	1	3	[11, 10]	10	[2, 2, 2, 2, 2, 2, 2, 2, 2]
14	[4, 0]	1	4	[11, 7]	4	[2, 3, 2, 2]
15	[5, 0]	1	5	[11, 8]	4	[2, 2, 3, 2]

## The Basic Curves on $\tilde{Z}_{11,2}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	12	12	0	-3
8	12	12	0	-3
9	8	8	0	-2
16	8	8	0	-2
17	1	1	0	-3
40	1	1	0	-3

**The intersection matrix for the non-exceptional curves:**

No	1	8	9	16	17	40
1	-3	162	0	110	0	30
8	162	-3	110	0	30	0
9	0	110	-2	72	0	20
16	110	0	72	-2	20	0
17	0	30	0	20	-3	5
40	30	0	20	0	5	-3

**The intersection matrix for the  $P_0$ -curves (curves 1...8)**

No	1	2	3	4	5	6	7	8
1	-3	1	1	1	1	1	1	162
2	1	-2	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	1
4	1	0	0	-2	0	0	0	1
5	1	0	0	0	-2	0	0	1
6	1	0	0	0	0	-2	0	1
7	1	0	0	0	0	0	-2	1
8	162	1	1	1	1	1	1	-3

**The intersection matrix for the  $P_1$ -curves (curves 9...16)**

No	9	10	11	12	13	14	15	16
9	-2	1	1	1	0	1	0	72
10	1	-3	0	0	0	0	0	1
11	1	0	-3	0	0	0	0	1
12	1	0	0	-2	1	0	0	0
13	0	0	0	1	-2	0	0	1
14	1	0	0	0	0	-2	1	0
15	0	0	0	0	0	1	-2	1
16	72	1	1	0	1	0	1	-2

The intersection matrix for the  $P_\infty$ -curves (curves 17...40)

No	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	-3	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1
18	1	-6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0
32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0
36	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	5	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0

No	37	38	39	40
17	0	0	0	5
18	0	0	0	0
19	0	0	0	1
20	0	0	0	0
21	0	0	0	1
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
25	0	0	0	0
26	0	0	0	0
27	0	0	0	0
28	0	0	0	0
29	0	0	0	0
30	0	0	0	0
31	0	0	0	1
32	0	0	0	0
33	0	0	0	0
34	0	0	0	0
35	0	0	0	1
36	1	0	0	0
37	-2	1	0	0
38	1	-3	1	0
39	0	1	-2	1
40	0	0	1	-3

The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{11,2}$  for  $n \leq 30$

Their basic properties:

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
41	2	5	3	0	0	0	-1
42	6	1	12	0	0	0	-2
43	7	2	8	0	0	0	-2
44	8	3	12	0	0	0	-2
45	10	4	18	0	0	0	-4
46	13	5	14	0	0	0	-2
47	17	1	18	1	1	0	-2
48	18	2	36	0	0	0	-4
49	19	3	20	1	1	0	-2
50	21	4	32	1	1	0	-2
51	24	5	48	1	1	0	-4
52	28	1	48	2	2	0	-4
53	29	2	30	2	2	0	-2
54	30	3	72	3	3	0	-2

Their intersection numbers with other curves:

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6	7	8
41	2	5	3	1	1	0	0	0	0	0	1
42	6	1	12	6	0	0	0	0	0	0	6
43	7	2	8	4	0	0	0	0	0	0	4
44	8	3	12	6	0	0	0	0	0	0	6
45	10	4	18	8	0	1	1	0	0	0	8
46	13	5	14	6	0	1	1	0	0	0	6
47	17	1	18	8	0	1	1	0	0	0	8
48	18	2	36	18	0	0	0	0	0	0	18
49	19	3	20	10	0	0	0	0	0	0	10
50	21	4	32	16	0	0	0	0	0	0	16
51	24	5	48	24	0	0	0	0	0	0	24
52	28	1	48	24	0	0	0	0	0	0	24
53	29	2	30	14	0	1	1	0	0	0	14
54	30	3	72	36	0	0	0	0	0	0	36

b) Those with the curves over  $P_1$ :

No	$n$	$k$	deg	9	10	11	12	13	14	15	16
41	2	5	3	1	0	0	0	0	0	0	1
42	6	1	12	4	0	0	0	0	0	0	4
43	7	2	8	2	1	1	0	0	0	0	2
44	8	3	12	4	0	0	0	0	0	0	4
45	10	4	18	6	0	0	0	0	0	0	6
46	13	5	14	4	1	1	0	0	0	0	4
47	17	1	18	6	0	0	0	0	0	0	6
48	18	2	36	12	0	0	0	0	0	0	12
49	19	3	20	6	1	1	0	0	0	0	6
50	21	4	32	10	1	1	0	0	0	0	10
51	24	5	48	16	0	0	0	0	0	0	16
52	28	1	48	16	0	0	0	0	0	0	16
53	29	2	30	10	0	0	0	0	0	0	10
54	30	3	72	24	0	0	0	0	0	0	24

c) Those with the curves over  $P_\infty$ :

No	$n$	$k$	deg	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
41	2	5	3	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	6	1	12	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
43	7	2	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
44	8	3	12	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
45	10	4	18	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0
46	13	5	14	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	17	1	18	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	18	2	36	1	2	0	0	2	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0
49	19	3	20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
50	21	4	32	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
51	24	5	48	2	1	1	1	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0
52	28	1	48	2	0	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0
53	29	2	30	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
54	30	3	72	3	2	0	0	2	0	0	0	0	1	1	0	0	0	0	1	1	0	1	1

No	$n$	$k$	deg	37	38	39	40
41	2	5	3	0	0	0	0
42	6	1	12	0	1	0	0
43	7	2	8	0	0	1	0
44	8	3	12	0	0	0	0
45	10	4	18	1	0	0	0
46	13	5	14	0	0	0	1
47	17	1	18	0	0	0	1
48	18	2	36	0	0	1	1
49	19	3	20	0	0	0	1
50	21	4	32	0	0	0	1
51	24	5	48	0	1	0	2
52	28	1	48	0	0	1	2
53	29	2	30	0	0	1	2
54	30	3	72	0	1	1	3

d) Those of the Hecke curves with each other:

No	$n$	$k$	deg	41	42	43	44	45	46	47	48	49	50	51	52	53	54
41	2	5	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
42	6	1	12	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
43	7	2	8	0	0	-2	0	0	0	0	0	0	0	0	0	0	0
44	8	3	12	0	0	0	-2	0	0	0	0	0	0	0	2	2	0
45	10	4	18	0	0	0	0	-4	0	0	0	0	0	0	0	0	0
46	13	5	14	0	0	0	0	0	-2	0	0	0	0	2	2	0	2
47	17	1	18	0	0	0	0	0	0	-2	2	2	0	0	2	2	4
48	18	2	36	0	0	0	0	0	0	2	-4	2	0	0	0	2	0
49	19	3	20	0	0	0	0	0	0	2	2	-2	2	4	2	0	2
50	21	4	32	0	0	0	0	0	0	0	0	2	-2	0	2	4	4
51	24	5	48	0	0	0	0	0	2	0	0	4	0	-4	0	6	2
52	28	1	48	0	0	0	2	0	2	2	0	2	2	0	-4	2	0
53	29	2	30	0	0	0	2	0	0	2	2	0	4	6	2	-2	4
54	30	3	72	0	0	0	0	0	2	4	0	2	4	2	0	4	-2