

Invariants of the Surface $\tilde{Z}_{12,11}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	6	62	74	-48	76	8

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2S_\infty$	$2S$
	576	25	8	11	6	7	0	8	0	14	38	58	-5	-5

The Singularities of the associated singular surface $Z_{12,11}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 7]	—	4	1	[12, 7], [5, 3]	[193, 1944, 4896]	[1, 0, 144]
2	[0, 11]	—	4	1	[12, 11], [1, 1]	[265, 552, 288]	[1, 0, 144]
3	[2, 9]	—	4	2	[2, 9], [-1, -4]	[85, -912, 2448]	[13, 10, 13]
4	[3, 8]	—	4	1	[3, 8], [1, 3]	[73, 648, 1440]	[9, 0, 16]
5	[3, 10]	—	4	2	[3, 10], [-1, -3]	[109, -792, 1440]	[13, 10, 13]
6	[4, 9]	—	4	1	[4, 9], [-1, -2]	[97, -528, 720]	[9, 0, 16]
7	[6, 7]	—	4	2	[6, 7], [-1, -1]	[85, -312, 288]	[4, 4, 37]
8	[6, 11]	—	4	2	[6, 11], [1, 2]	[157, 672, 720]	[4, 4, 37]

The CM-Singularities above P_1 (those of type (-3))

– there are none of this type

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[0, 1]	—	6	1	[0, 1], [-1, 0]	[1, -12, 144]	[1, 0, 108]
10	[0, 5]	—	6	1	[12, 5], [-5, -2]	[229, -2268, 5616]	[1, 0, 108]
11	[1, 3]	—	6	1	[1, 3], [0, 1]	[13, 84, 144]	[9, -6, 13]
12	[3, 1]	—	6	1	[3, 1], [-1, 0]	[13, -84, 144]	[9, 6, 13]
13	[3, 4]	—	6	1	[3, 4], [-1, -1]	[37, -252, 432]	[9, -6, 13]
14	[4, 3]	—	6	1	[4, 3], [1, 1]	[37, 252, 432]	[9, 6, 13]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[12, 11]	11	[2, 2, 2, 2, 2, 2, 2, 2, 2]
16	[1, 2]	1	2	[2, 1]	1	[2]
17	[1, 3]	2	3	[3, 2]	2	[2, 2]
18	[1, 4]	2	4	[4, 3]	3	[2, 2, 2]
19	[1, 6]	1	5	[6, 5]	5	[2, 2, 2, 2, 2]
20	[1, 8]	2	4	[4, 3]	3	[2, 2, 2]
21	[1, 9]	2	3	[3, 2]	2	[2, 2]
22	[5, 0]	1	6	[12, 11]	11	[2, 2, 2, 2, 2, 2, 2, 2, 2]

The Basic Curves on $\tilde{Z}_{12,11}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	11	11	0	-4
10	11	11	0	-4
11	7	7	0	-4
24	7	7	0	-4
25	0	0	0	-6
64	0	0	0	-6

The intersection matrix for the non-exceptional curves:

No	1	10	11	24	25	64
1	-4	140	0	96	0	24
10	140	-4	96	0	24	0
11	0	96	-4	62	0	16
24	96	0	62	-4	16	0
25	0	24	0	16	-6	2
64	24	0	16	0	2	-6

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	140
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	140	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...24)

No	11	12	13	14	15	16	17	18	19	20	21	22	23	24
11	-4	1	0	1	0	1	0	1	0	1	0	1	0	62
12	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
13	0	1	-2	0	0	0	0	0	0	0	0	0	0	1
14	1	0	0	-2	1	0	0	0	0	0	0	0	0	0
15	0	0	0	1	-2	0	0	0	0	0	0	0	0	1
16	1	0	0	0	0	-2	1	0	0	0	0	0	0	0
17	0	0	0	0	0	1	-2	0	0	0	0	0	0	1
18	1	0	0	0	0	0	0	-2	1	0	0	0	0	0
19	0	0	0	0	0	0	0	1	-2	0	0	0	0	1
20	1	0	0	0	0	0	0	0	0	-2	1	0	0	0
21	0	0	0	0	0	0	0	0	0	1	-2	0	0	1
22	1	0	0	0	0	0	0	0	0	0	0	-2	1	0
23	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
24	62	0	1	0	1	0	1	0	1	0	1	0	1	-4

The intersection matrix for the P_∞ -curves (curves 25...64)

No	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
25	-6	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	0
26	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0
37	1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0
38	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0
43	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	2	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	0	0

No	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
25	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	2
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
48	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	1
51	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	1
53	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
64	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	-6

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{12,11}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
65	11	1	12	1	1	0	-2
66	11	5	12	1	1	0	-2
67	23	1	24	2	2	0	-2
68	23	5	24	2	2	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
65	11	1	12	6	0	0	0	0	0	0	0	0	6
66	11	5	12	6	0	0	0	0	0	0	0	0	6
67	23	1	24	12	0	0	0	0	0	0	0	0	12
68	23	5	24	12	0	0	0	0	0	0	0	0	12

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18	19	20	21	22	23	24
65	11	1	12	4	0	0	0	0	0	0	0	0	0	0	0	0	4
66	11	5	12	4	0	0	0	0	0	0	0	0	0	0	0	0	4
67	23	1	24	8	0	0	0	0	0	0	0	0	0	0	0	0	8
68	23	5	24	8	0	0	0	0	0	0	0	0	0	0	0	0	8

c) Those with the curves over P_∞ :

No	n	k	deg	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
65	11	1	12	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
66	11	5	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	23	1	24	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
68	23	5	24	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

No	n	k	deg	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
65	11	1	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	11	5	12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
67	23	1	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
68	23	5	24	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1

d) Those of the Hecke curves with each other:

No	n	k	deg	65	66	67	68
65	11	1	12	-2	0	0	2
66	11	5	12	0	-2	2	0
67	23	1	24	0	2	-2	0
68	23	5	24	2	0	0	-2