

Invariants of the Surface $\tilde{Z}_{12,1}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	1	36	38	-32	40	-16

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	576	25	8	11	6	7	6	8	0	14	8	22	4	5

The Singularities of the associated singular surface $Z_{12,1}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 7]	+	1	1	[12, 7], [5, 3]	[193, 1944, 4896]	[1, 0, 144]
2	[0, 11]	+	1	2	[12, 11], [1, 1]	[265, 552, 288]	[1, 0, 144]
3	[2, 9]	+	2	3	[2, 9], [-1, -4]	[85, -912, 2448]	[13, 10, 13]
4	[3, 8]	+	2	4	[3, 8], [1, 3]	[73, 648, 1440]	[9, 0, 16]
5	[3, 10]	+	2	3	[3, 10], [-1, -3]	[109, -792, 1440]	[13, 10, 13]
6	[4, 9]	+	2	4	[4, 9], [-1, -2]	[97, -528, 720]	[9, 0, 16]
7	[6, 7]	+	1	5	[6, 7], [-1, -1]	[85, -312, 288]	[4, 4, 37]
8	[6, 11]	+	1	6	[6, 11], [1, 2]	[157, 672, 720]	[4, 4, 37]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -12, 144]	[1, 0, 108]
10	[0, 5]	+	1	2	[12, 5], [-5, -2]	[229, -2268, 5616]	[1, 0, 108]
11	[1, 3]	+	2	3	[1, 3], [0, 1]	[13, 84, 144]	[9, -6, 13]
12	[3, 1]	+	2	3	[3, 1], [-1, 0]	[13, -84, 144]	[9, 6, 13]
13	[3, 4]	+	2	4	[3, 4], [-1, -1]	[37, -252, 432]	[9, -6, 13]
14	[4, 3]	+	2	4	[4, 3], [1, 1]	[37, 252, 432]	[9, 6, 13]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

– there are none of this type

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[12, 1]	1	[12]
16	[1, 2]	1	2	[2, 1]	1	[2]
17	[1, 3]	2	3	[3, 1]	1	[3]
18	[1, 4]	2	4	[4, 1]	1	[4]
19	[1, 6]	1	5	[6, 1]	1	[6]
20	[1, 8]	2	4	[4, 1]	1	[4]
21	[1, 9]	2	3	[3, 1]	1	[3]
22	[5, 0]	1	6	[12, 1]	1	[12]

The Basic Curves on $\tilde{Z}_{12,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	11	11	0	-4
10	11	11	0	-4
11	7	7	0	-2
18	7	7	0	-2
19	0	0	0	-2
28	0	0	0	-2

The intersection matrix for the non-exceptional curves:

No	1	10	11	18	19	28
1	-4	140	0	96	0	24
10	140	-4	96	0	24	0
11	0	96	-2	62	0	16
18	96	0	62	-2	16	0
19	0	24	0	16	-2	2
28	24	0	16	0	2	-2

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	140
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	140	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...18)

No	11	12	13	14	15	16	17	18
11	-2	1	1	1	1	1	1	62
12	1	-3	0	0	0	0	0	1
13	1	0	-3	0	0	0	0	1
14	1	0	0	-3	0	0	0	1
15	1	0	0	0	-3	0	0	1
16	1	0	0	0	0	-3	0	1
17	1	0	0	0	0	0	-3	1
18	62	1	1	1	1	1	1	-2

The intersection matrix for the P_∞ -curves (curves 19...28)

No	19	20	21	22	23	24	25	26	27	28
19	-2	1	1	1	1	1	1	1	1	2
20	1	-12	0	0	0	0	0	0	0	1
21	1	0	-2	0	0	0	0	0	0	1
22	1	0	0	-3	0	0	0	0	0	1
23	1	0	0	0	-4	0	0	0	0	1
24	1	0	0	0	0	-6	0	0	0	1
25	1	0	0	0	0	0	-4	0	0	1
26	1	0	0	0	0	0	0	-3	0	1
27	1	0	0	0	0	0	0	0	-12	1
28	2	1	1	1	1	1	1	1	1	-2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{12,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
29	1	1	1	0	0	0	-1
30	1	5	1	0	0	0	-1
31	13	1	14	0	0	0	-2
32	13	5	14	0	0	0	-2
33	25	1	30	0	0	0	-2
34	25	5	30	0	0	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
29	1	1	1	0	0	1	0	0	0	0	0	0	0
30	1	5	1	0	1	0	0	0	0	0	0	0	0
31	13	1	14	6	0	0	1	0	1	0	0	0	6
32	13	5	14	6	0	0	1	0	1	0	0	0	6
33	25	1	30	14	0	0	0	1	0	1	0	0	14
34	25	5	30	14	0	0	0	1	0	1	0	0	14

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18
29	1	1	1	0	1	0	0	0	0	0	0
30	1	5	1	0	0	1	0	0	0	0	0
31	13	1	14	4	0	0	1	1	0	0	4
32	13	5	14	4	0	0	0	0	1	1	4
33	25	1	30	10	0	0	0	0	0	0	10
34	25	5	30	10	0	0	0	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	19	20	21	22	23	24	25	26	27	28
29	1	1	1	0	1	0	0	0	0	0	0	0	0
30	1	5	1	0	0	0	0	0	0	0	0	1	0
31	13	1	14	1	2	0	0	0	0	0	0	0	1
32	13	5	14	1	0	0	0	0	0	0	0	2	1
33	25	1	30	2	2	0	0	0	0	0	0	4	2
34	25	5	30	2	4	0	0	0	0	0	0	2	2

d) Those of the Hecke curves with each other:

No	n	k	deg	29	30	31	32	33	34
29	1	1	1	-1	0	0	0	0	0
30	1	5	1	0	-1	0	0	0	0
31	13	1	14	0	0	-2	0	2	0
32	13	5	14	0	0	0	-2	0	2
33	25	1	30	0	0	2	0	-2	2
34	25	5	30	0	0	0	2	2	-2