

Invariants of the Surface $\tilde{Z}_{12,5}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	4	46	54	-36	56	4

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2S_\infty$	$2S$
	576	25	8	11	6	7	0	8	0	14	18	38	-1	-1

The Singularities of the associated singular surface $Z_{12,5}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 8]	+	2	1	[1, 8], [0, 1]	[65, 192, 144]	[9, -6, 17]
2	[1, 10]	+	2	2	[1, 10], [0, 1]	[101, 240, 144]	[5, 2, 29]
3	[2, 7]	+	2	3	[2, 7], [-1, -3]	[53, -552, 1440]	[5, 2, 29]
4	[2, 11]	+	2	2	[2, 11], [-1, -5]	[125, -1368, 3744]	[5, -2, 29]
5	[4, 7]	+	2	4	[4, 7], [1, 2]	[65, 432, 720]	[9, -6, 17]
6	[4, 11]	+	2	1	[4, 11], [1, 3]	[137, 888, 1440]	[9, 6, 17]
7	[5, 8]	+	2	4	[5, 8], [-2, -3]	[89, -816, 1872]	[9, 6, 17]
8	[5, 10]	+	2	3	[5, 22], [2, 9]	[509, 4992, 12240]	[5, -2, 29]

The CM-Singularities above P_1 (those of type (-3))

- there are none of this type

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[1, 2]	-	6	1	[1, 2], [0, 1]	[7, 60, 144]	[7, 4, 16]
10	[1, 5]	-	6	1	[1, 5], [0, 1]	[31, 132, 144]	[4, 0, 27]
11	[1, 6]	-	6	1	[1, 6], [0, 1]	[43, 156, 144]	[4, 0, 27]
12	[2, 1]	-	6	1	[2, 1], [-1, 0]	[7, -60, 144]	[7, -4, 16]
13	[2, 3]	-	6	1	[2, 3], [-1, -1]	[19, -180, 432]	[7, 4, 16]
14	[3, 2]	-	6	1	[3, 2], [1, 1]	[19, 180, 432]	[7, -4, 16]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[12, 5]	3	[3, 2, 3]
16	[1, 2]	1	2	[2, 1]	1	[2]
17	[1, 3]	2	3	[3, 2]	2	[2, 2]
18	[1, 4]	2	4	[4, 1]	1	[4]
19	[1, 6]	1	5	[6, 5]	5	[2, 2, 2, 2, 2]
20	[1, 8]	2	4	[4, 1]	1	[4]
21	[1, 9]	2	3	[3, 2]	2	[2, 2]
22	[5, 0]	1	6	[12, 5]	3	[3, 2, 3]

The Basic Curves on $\tilde{Z}_{12,5}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	11	11	0	-4
10	11	11	0	-4
11	7	7	0	-4
24	7	7	0	-4
25	0	0	0	-4
44	0	0	0	-4

The intersection matrix for the non-exceptional curves:

No	1	10	11	24	25	44
1	-4	140	0	96	0	24
10	140	-4	96	0	24	0
11	0	96	-4	62	0	16
24	96	0	62	-4	16	0
25	0	24	0	16	-4	2
44	24	0	16	0	2	-4

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	140
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	140	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...24)

No	11	12	13	14	15	16	17	18	19	20	21	22	23	24
11	-4	1	0	1	0	1	0	1	0	1	0	1	0	62
12	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
13	0	1	-2	0	0	0	0	0	0	0	0	0	0	1
14	1	0	0	-2	1	0	0	0	0	0	0	0	0	0
15	0	0	0	1	-2	0	0	0	0	0	0	0	0	1
16	1	0	0	0	0	-2	1	0	0	0	0	0	0	0
17	0	0	0	0	0	1	-2	0	0	0	0	0	0	1
18	1	0	0	0	0	0	0	-2	1	0	0	0	0	0
19	0	0	0	0	0	0	0	1	-2	0	0	0	0	1
20	1	0	0	0	0	0	0	0	0	-2	1	0	0	0
21	0	0	0	0	0	0	0	0	0	1	-2	0	0	1
22	1	0	0	0	0	0	0	0	0	0	0	-2	1	0
23	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
24	62	0	1	0	1	0	1	0	1	0	1	0	1	-4

The intersection matrix for the P_∞ -curves (curves 25...44)

No	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
25	-4	1	0	0	1	1	0	1	1	0	0	0	0	1	1	0	1	0	0	2
26	1	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
29	1	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
30	1	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1
32	1	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	1
33	1	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	1
38	1	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	1
39	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	1
41	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	1
44	2	0	0	1	1	0	1	1	0	0	0	0	1	1	0	1	0	0	1	-4

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{12,5}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
45	5	1	6	0	0	0	-2
46	5	5	6	0	0	0	-2
47	17	1	18	1	1	0	-2
48	17	5	18	1	1	0	-2
49	29	1	30	2	2	0	-2
50	29	5	30	2	2	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
45	5	1	6	2	0	0	1	0	0	0	0	1	2
46	5	5	6	2	0	1	0	1	0	0	0	0	2
47	17	1	18	8	0	0	0	0	1	0	1	0	8
48	17	5	18	8	1	0	0	0	0	1	0	0	8
49	29	1	30	14	0	1	0	1	0	0	0	0	14
50	29	5	30	14	0	0	1	0	0	0	0	1	14

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18	19	20	21	22	23	24
45	5	1	6	2	0	0	0	0	0	0	0	0	0	0	0	0	2
46	5	5	6	2	0	0	0	0	0	0	0	0	0	0	0	0	2
47	17	1	18	6	0	0	0	0	0	0	0	0	0	0	0	0	6
48	17	5	18	6	0	0	0	0	0	0	0	0	0	0	0	0	6
49	29	1	30	10	0	0	0	0	0	0	0	0	0	0	0	0	10
50	29	5	30	10	0	0	0	0	0	0	0	0	0	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
45	5	1	6	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
46	5	5	6	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
47	17	1	18	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
48	17	5	18	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
49	29	1	30	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2
50	29	5	30	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2

d) Those of the Hecke curves with each other:

No	n	k	deg	45	46	47	48	49	50
45	5	1	6	-2	0	0	0	0	0
46	5	5	6	0	-2	0	0	0	0
47	17	1	18	0	0	-2	2	2	0
48	17	5	18	0	0	2	-2	0	2
49	29	1	30	0	0	2	0	-2	4
50	29	5	30	0	0	0	2	4	-2