

Invariants of the Surface $\tilde{Z}_{12,7}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	3	40	46	-32	48	0

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2S_\infty$	$2S$
	576	25	8	11	6	7	6	8	0	14	16	30	0	1

The Singularities of the associated singular surface $Z_{12,7}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 8]	-	4	1	[1, 8], [0, 1]	[65, 192, 144]	[9, -6, 17]
2	[1, 10]	-	4	2	[1, 10], [0, 1]	[101, 240, 144]	[5, 2, 29]
3	[2, 7]	-	4	2	[2, 7], [-1, -3]	[53, -552, 1440]	[5, 2, 29]
4	[2, 11]	-	4	2	[2, 11], [-1, -5]	[125, -1368, 3744]	[5, -2, 29]
5	[4, 7]	-	4	1	[4, 7], [1, 2]	[65, 432, 720]	[9, -6, 17]
6	[4, 11]	-	4	1	[4, 11], [1, 3]	[137, 888, 1440]	[9, 6, 17]
7	[5, 8]	-	4	1	[5, 8], [-2, -3]	[89, -816, 1872]	[9, 6, 17]
8	[5, 10]	-	4	2	[5, 22], [2, 9]	[509, 4992, 12240]	[5, -2, 29]

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[1, 2]	+	2	1	[1, 2], [0, 1]	[7, 60, 144]	[7, 4, 16]
10	[1, 5]	+	2	2	[1, 5], [0, 1]	[31, 132, 144]	[4, 0, 27]
11	[1, 6]	+	2	2	[1, 6], [0, 1]	[43, 156, 144]	[4, 0, 27]
12	[2, 1]	+	2	1	[2, 1], [-1, 0]	[7, -60, 144]	[7, -4, 16]
13	[2, 3]	+	2	3	[2, 3], [-1, -1]	[19, -180, 432]	[7, 4, 16]
14	[3, 2]	+	2	3	[3, 2], [1, 1]	[19, 180, 432]	[7, -4, 16]

The anti-CM-Singularities above P1 (those of type (-2, -2))

- there are none of this type

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[12, 7]	3	[2, 4, 2]
16	[1, 2]	1	2	[2, 1]	1	[2]
17	[1, 3]	2	3	[3, 1]	1	[3]
18	[1, 4]	2	4	[4, 3]	3	[2, 2, 2]
19	[1, 6]	1	5	[6, 1]	1	[6]
20	[1, 8]	2	4	[4, 3]	3	[2, 2, 2]
21	[1, 9]	2	3	[3, 1]	1	[3]
22	[5, 0]	1	6	[12, 7]	3	[2, 4, 2]

The Basic Curves on $\tilde{Z}_{12,7}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	11	11	0	-4
10	11	11	0	-4
11	7	7	0	-2
18	7	7	0	-2
19	0	0	0	-4
36	0	0	0	-4

The intersection matrix for the non-exceptional curves:

No	1	10	11	18	19	36
1	-4	140	0	96	0	24
10	140	-4	96	0	24	0
11	0	96	-2	62	0	16
18	96	0	62	-2	16	0
19	0	24	0	16	-4	2
36	24	0	16	0	2	-4

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	140
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	140	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...18)

No	11	12	13	14	15	16	17	18
11	-2	1	1	1	1	1	1	62
12	1	-3	0	0	0	0	0	1
13	1	0	-3	0	0	0	0	1
14	1	0	0	-3	0	0	0	1
15	1	0	0	0	-3	0	0	1
16	1	0	0	0	0	-3	0	1
17	1	0	0	0	0	0	-3	1
18	62	1	1	1	1	1	1	-2

The intersection matrix for the P_∞ -curves (curves 19...36)

No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
19	-4	1	0	0	1	1	1	0	0	1	1	0	0	1	1	0	0	2
20	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	1	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	1
24	1	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	1
25	1	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	1
28	1	0	0	0	0	0	0	0	0	-6	0	0	0	0	0	0	0	1
29	1	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	1
32	1	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	1
33	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
36	2	0	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	-4

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{12,7}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
37	7	1	8	0	0	0	-2
38	7	5	8	0	0	0	-2
39	19	1	20	1	1	0	-2
40	19	5	20	1	1	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
37	7	1	8	4	0	0	0	0	0	0	0	0	4
38	7	5	8	4	0	0	0	0	0	0	0	0	4
39	19	1	20	10	0	0	0	0	0	0	0	0	10
40	19	5	20	10	0	0	0	0	0	0	0	0	10

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18
37	7	1	8	2	0	0	0	0	1	1	2
38	7	5	8	2	1	0	0	1	0	0	2
39	19	1	20	6	1	0	0	1	0	0	6
40	19	5	20	6	0	0	0	0	1	1	6

c) Those with the curves over P_∞ :

No	n	k	deg	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	7	1	8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
38	7	5	8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
39	19	1	20	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1
40	19	5	20	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

d) Those of the Hecke curves with each other:

No	n	k	deg	37	38	39	40
37	7	1	8	-2	0	0	0
38	7	5	8	0	-2	0	0
39	19	1	20	0	0	-2	0
40	19	5	20	0	0	0	-2