

# Invariants of the Surface $\tilde{Z}_{13,2}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	4	44	52	-34	54	6

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$L_\infty$	$L$	$2S_\infty$	$2S$
	1092	50	6	24	4	16	2	6	2	16	22	34	0	0

## The Singularities of the associated singular surface $Z_{13,2}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[1, 12]	+	1	1	[1, 12], [0, 1]	[145, 312, 169]	[2, 2, 85]
2	[4, 8]	+	2	2	[4, 21], [-1, -5]	[457, -2834, 4394]	[5, 2, 34]
3	[5, 9]	+	2	2	[5, 9], [1, 2]	[106, 598, 845]	[5, -2, 34]
4	[1, 7]	-	3	3	[1, 7], [0, 1]	[50, 182, 169]	[5, -2, 34]
5	[5, 8]	-	3	3	[5, 8], [-2, -3]	[89, -884, 2197]	[2, 2, 85]
6	[6, 12]	-	3	3	[6, 25], [-1, -4]	[661, -2756, 2873]	[5, 2, 34]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
7	[2, 4]	+	2	1	[2, 17], [-1, -8]	[327, -4017, 12337]	[7, -5, 19]
8	[4, 2]	+	2	1	[4, 15], [1, 4]	[301, 2067, 3549]	[7, 5, 19]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
9	[3, 4]	-	2	2	[3, 4], [-1, -1]	[37, -273, 507]	[7, 5, 19]
10	[4, 3]	-	2	2	[4, 3], [1, 1]	[37, 273, 507]	[7, -5, 19]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	[1, 0]	1	1	[13, 2]	2	[7, 2]
12	[2, 0]	1	2	[13, 7]	2	[2, 7]
13	[3, 0]	1	3	[13, 6]	6	[3, 2, 2, 2, 2, 2]
14	[4, 0]	1	4	[13, 5]	3	[3, 3, 2]
15	[5, 0]	1	5	[13, 11]	6	[2, 2, 2, 2, 2, 3]
16	[6, 0]	1	6	[13, 8]	3	[2, 3, 3]

## The Basic Curves on $\tilde{Z}_{13,2}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	24	24	0	-3
8	24	24	0	-3
9	16	16	0	-2
16	16	16	0	-2
17	2	2	0	-3
40	2	2	0	-3

**The intersection matrix for the non-exceptional curves:**

No	1	8	9	16	17	40
1	-3	270	0	182	0	42
8	270	-3	182	0	42	0
9	0	182	-2	120	0	28
16	182	0	120	-2	28	0
17	0	42	0	28	-3	6
40	42	0	28	0	6	-3

**The intersection matrix for the  $P_0$ -curves (curves 1...8)**

No	1	2	3	4	5	6	7	8
1	-3	1	1	1	1	1	1	270
2	1	-2	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	1
4	1	0	0	-2	0	0	0	1
5	1	0	0	0	-2	0	0	1
6	1	0	0	0	0	-2	0	1
7	1	0	0	0	0	0	-2	1
8	270	1	1	1	1	1	1	-3

**The intersection matrix for the  $P_1$ -curves (curves 9...16)**

No	9	10	11	12	13	14	15	16
9	-2	1	1	1	0	1	0	120
10	1	-3	0	0	0	0	0	1
11	1	0	-3	0	0	0	0	1
12	1	0	0	-2	1	0	0	0
13	0	0	0	1	-2	0	0	1
14	1	0	0	0	0	-2	1	0
15	0	0	0	0	0	1	-2	1
16	120	1	1	0	1	0	1	-2

The intersection matrix for the  $P_\infty$ -curves (curves 17...40)

No	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	-3	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
18	1	-7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0
28	1	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	1	-3	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0
31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3
37	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	6	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1

No	37	38	39	40
17	1	0	0	6
18	0	0	0	0
19	0	0	0	1
20	0	0	0	0
21	0	0	0	1
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
25	0	0	0	0
26	0	0	0	0
27	0	0	0	1
28	0	0	0	0
29	0	0	0	0
30	0	0	0	1
31	0	0	0	0
32	0	0	0	0
33	0	0	0	0
34	0	0	0	0
35	0	0	0	0
36	0	0	0	1
37	-2	1	0	0
38	1	-3	1	0
39	0	1	-3	1
40	0	0	1	-3

The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{13,2}$  for  $n \leq 30$

**Their basic properties:**

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
41	2	6	3	0	0	0	-1
42	5	2	6	0	0	0	-2
43	6	5	12	0	0	0	-2
44	7	1	8	0	0	0	-2
45	8	3	12	0	0	0	-2
46	11	4	12	1	1	0	-2
47	15	6	24	1	1	0	-2
48	18	2	36	0	0	0	-4
49	19	5	20	1	1	0	-2
50	20	1	36	1	1	0	-4
51	21	3	32	1	1	0	-4
52	24	4	48	1	1	0	-4
53	28	6	48	2	2	0	-2

**Their intersection numbers with other curves:**

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6	7	8
41	2	6	3	1	1	0	0	0	0	0	1
42	5	2	6	2	0	1	1	0	0	0	2
43	6	5	12	6	0	0	0	0	0	0	6
44	7	1	8	4	0	0	0	0	0	0	4
45	8	3	12	6	0	0	0	0	0	0	6
46	11	4	12	6	0	0	0	0	0	0	6
47	15	6	24	12	0	0	0	0	0	0	12
48	18	2	36	18	0	0	0	0	0	0	18
49	19	5	20	10	0	0	0	0	0	0	10
50	20	1	36	18	0	0	0	0	0	0	18
51	21	3	32	16	0	0	0	0	0	0	16
52	24	4	48	24	0	0	0	0	0	0	24
53	28	6	48	24	0	0	0	0	0	0	24

b) Those with the curves over  $P_1$ :

No	$n$	$k$	deg	9	10	11	12	13	14	15	16
41	2	6	3	1	0	0	0	0	0	0	1
42	5	2	6	2	0	0	0	0	0	0	2
43	6	5	12	4	0	0	0	0	0	0	4
44	7	1	8	2	1	1	0	0	0	0	2
45	8	3	12	4	0	0	0	0	0	0	4
46	11	4	12	4	0	0	0	0	0	0	4
47	15	6	24	8	0	0	0	0	0	0	8
48	18	2	36	12	0	0	0	0	0	0	12
49	19	5	20	6	1	1	0	0	0	0	6
50	20	1	36	12	0	0	0	0	0	0	12
51	21	3	32	10	1	1	0	0	0	0	10
52	24	4	48	16	0	0	0	0	0	0	16
53	28	6	48	16	0	0	0	0	0	0	16

c) Those with the curves over  $P_\infty$ :

No	$n$	$k$	deg	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
41	2	6	3	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	5	2	6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
43	6	5	12	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1
44	7	1	8	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	8	3	12	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
46	11	4	12	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
47	15	6	24	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
48	18	2	36	1	2	0	0	2	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0
49	19	5	20	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
50	20	1	36	1	0	1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0
51	21	3	32	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
52	24	4	48	1	1	1	1	1	1	0	0	0	0	1	0	1	0	1	0	0	0	0	1
53	28	6	48	2	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0

No	$n$	$k$	deg	37	38	39	40
41	2	6	3	0	0	0	0
42	5	2	6	0	0	1	0
43	6	5	12	0	1	0	0
44	7	1	8	0	0	0	0
45	8	3	12	1	0	0	0
46	11	4	12	0	0	0	0
47	15	6	24	0	0	0	1
48	18	2	36	0	0	1	1
49	19	5	20	0	0	0	1
50	20	1	36	0	0	1	1
51	21	3	32	1	0	0	1
52	24	4	48	0	1	0	1
53	28	6	48	0	1	1	2

d) Those of the Hecke curves with each other:

No	$n$	$k$	deg	41	42	43	44	45	46	47	48	49	50	51	52	53
41	2	6	3	-1	0	0	0	0	0	0	0	0	0	0	0	0
42	5	2	6	0	-2	0	0	0	0	0	0	0	0	0	0	0
43	6	5	12	0	0	-2	0	0	0	0	0	0	0	0	0	0
44	7	1	8	0	0	0	-2	0	0	0	0	0	0	0	0	0
45	8	3	12	0	0	0	0	-2	0	0	0	0	0	0	0	0
46	11	4	12	0	0	0	0	0	-2	0	0	0	0	0	0	2
47	15	6	24	0	0	0	0	0	0	-2	0	0	0	0	0	2
48	18	2	36	0	0	0	0	0	0	0	-4	2	0	0	0	0
49	19	5	20	0	0	0	0	0	0	0	2	-2	0	0	0	2
50	20	1	36	0	0	0	0	0	0	0	0	0	-4	0	0	0
51	21	3	32	0	0	0	0	0	0	0	0	0	0	-4	0	2
52	24	4	48	0	0	0	0	0	0	0	0	0	0	0	-4	0
53	28	6	48	0	0	0	0	0	2	2	0	2	0	2	0	-2