Invariants of the Surface $\tilde{Z}_{14,3}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	6	59	71	-45	73	11

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	1008	49	8	23	6	15	3	9	1	20	32	49	-2	-2

The Singularities of the associated singular surface $Z_{14,3}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	$[1,10]$	$+$	2	1	$[1,10],[0,1]$	$[101,280,196]$	$[13,10,17]$
2	$[3,8]$	$+$	2	2	$[3,8],[1,3]$	$[73,756,1960]$	$[5,4,40]$
3	$[4,13]$	$+$	2	1	$[4,13],[-1,-3]$	$[185,-1204,1960]$	$[13,-10,17]$
4	$[6,11]$	$+$	2	2	$[6,11],[1,2]$	$[157,784,980]$	$[5,-4,40]$
5	$[0,9]$	$-$	4	3	$[14,9],[3,2]$	$[277,1680,2548]$	$[1,0,196]$
6	$[3,10]$	$-$	4	3	$[3,10],[-1,-3]$	$[109,-924,1960]$	$[8,4,25]$
7	$[4,11]$	$-$	4	3	$[4,11],[1,3]$	$[137,1036,1960]$	$[8,-4,25]$
8	$[7,12]$	$-$	4	3	$[7,12],[-3,-5]$	$[193,-2268,6664]$	$[4,0,49]$

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	$[1,1]$	$+$	1	1	$[1,1],[-1,0]$	$[3,-42,196]$	$[3,0,49]$
10	$[1,5]$	$+$	2	2	$[1,5],[0,1]$	$[31,154,196]$	$[12,-6,13]$
11	$[1,8]$	$+$	2	2	$[1,8],[0,1]$	$[73,238,196]$	$[12,6,13]$

The anti-CM-Singularities above P1 (those of type $(-2,-2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
12	$[0,5]$	$-$	3	3	$[14,5],[-3,-1]$	$[291,-1722,2548]$	$[1,0,147]$
13	$[2,5]$	$-$	3	3	$[2,5],[-1,-2]$	$[39,-462,1372]$	$[4,2,37]$
14	$[2,7]$	$-$	3	3	$[2,7],[-1,-3]$	$[67,-826,2548]$	$[4,-2,37]$

The Singularities above P_∞:

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	$[1,0]$	1	1	$[14,3]$	2	$[5,3]$
16	$[1,2]$	3	2	$[2,1]$	1	$[2]$
17	$[1,4]$	3	2	$[2,1]$	1	$[2]$
18	$[1,6]$	3	2	$[2,1]$	1	$[2]$
19	$[1,7]$	1	3	$[7,3]$	3	$[3,2,2]$
20	$[2,7]$	1	4	$[7,6]$	6	$[2,2,2,2,2,2]$
21	$[3,0]$	1	5	$[14,5]$	2	$[3,5]$
22	$[3,7]$	1	6	$[7,5]$	3	$[2,2,3]$
23	$[5,0]$	1	7	$[14,13]$	13	$[2,2,2,2,2,2,2,2,2,2,2,2,2]$

The Basic Curves on $\tilde{Z}_{14,3}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	23	23	0	-4
10	23	23	0	-4
11	15	15	0	-3
21	15	15	0	-3
22	1	1	0	-5
55	1	1	0	-5

The intersection matrix for the non-exceptional curves:

No	1	10	11	21	22	55
1	-4	248	0	168	0	36
10	248	-4	168	0	36	0
11	0	168	-3	110	0	24
21	168	0	110	-3	24	0
22	0	36	0	24	-5	3
55	36	0	24	0	3	-5

The intersection matrix for the P_0-curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	248
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	248	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1-curves (curves 11...21)

No	11	12	13	14	15	16	17	18	19	20	21
11	-3	1	1	1	1	0	1	0	1	0	110
12	1	-3	0	0	0	0	0	0	0	0	1
13	1	0	-3	0	0	0	0	0	0	0	1
14	1	0	0	-3	0	0	0	0	0	0	1
15	1	0	0	0	-2	1	0	0	0	0	0
16	0	0	0	0	1	-2	0	0	0	0	1
17	1	0	0	0	0	0	-2	1	0	0	0
18	0	0	0	0	0	0	1	-2	0	0	1
19	1	0	0	0	0	0	0	0	-2	1	0
20	0	0	0	0	0	0	0	0	1	-2	1
21	110	1	1	1	0	1	0	1	0	1	-3

The intersection matrix for the P_∞-curves (curves 22...55)

No	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
22	−5	1	0	1	1	1	1	0	0	1	0	0	0	0	0	1	0	1	0	0
23	1	−5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	1	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	1	0	0	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	0	0	0	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	1	−2	0	0	0	0	0	0	0	0	0	0	0
31	1	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	0	0	0	0	0
37	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−5	0	0	0
39	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−3
42	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	3	0	1	1	1	1	0	0	1	0	0	0	0	0	1	0	1	0	0	1

No	42	43	44	45	46	47	48	49	50	51	52	53	54	55
22	1	0	0	0	0	0	0	0	0	0	0	0	0	3
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	1
26	0	0	0	0	0	0	0	0	0	0	0	0	0	1
27	0	0	0	0	0	0	0	0	0	0	0	0	0	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	1
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	1
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	1
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	−2	1	0	0	0	0	0	0	0	0	0	0	0	0
43	1	−2	1	0	0	0	0	0	0	0	0	0	0	0
44	0	1	−2	1	0	0	0	0	0	0	0	0	0	0
45	0	0	1	−2	1	0	0	0	0	0	0	0	0	0
46	0	0	0	1	−2	1	0	0	0	0	0	0	0	0
47	0	0	0	0	1	−2	1	0	0	0	0	0	0	0
48	0	0	0	0	0	1	−2	1	0	0	0	0	0	0
49	0	0	0	0	0	0	1	−2	1	0	0	0	0	0
50	0	0	0	0	0	0	0	1	−2	1	0	0	0	0
51	0	0	0	0	0	0	0	0	1	−2	1	0	0	0
52	0	0	0	0	0	0	0	0	0	1	−2	1	0	0
53	0	0	0	0	0	0	0	0	0	0	1	−2	1	0
54	0	0	0	0	0	0	0	0	0	0	0	1	−2	1
55	0	0	0	0	0	0	0	0	0	0	0	0	1	−5

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{14,3}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
56	3	5	4	0	0	0	−1
57	5	1	6	0	0	0	−2
58	13	3	14	0	0	0	−4
59	17	5	18	1	1	0	−2
60	19	1	20	1	1	0	−2
61	27	3	36	1	1	0	−4

Their intersection numbers with other curves:

a) Those with the curves over P_0:

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
56	3	5	4	2	0	0	0	0	0	0	0	0	2
57	5	1	6	2	0	1	0	1	0	0	0	0	2
58	13	3	14	6	1	0	1	0	0	0	0	0	6
59	17	5	18	8	1	0	1	0	0	0	0	0	8
60	19	1	20	10	0	0	0	0	0	0	0	0	10
61	27	3	36	18	0	0	0	0	0	0	0	0	18

b) Those with the curves over P_1:

No	n	k	deg	11	12	13	14	15	16	17	18	19	20	21
56	3	5	4	1	1	0	0	0	0	0	0	0	0	1
57	5	1	6	2	0	0	0	0	0	0	0	0	0	2
58	13	3	14	4	0	1	1	0	0	0	0	0	0	4
59	17	5	18	6	0	0	0	0	0	0	0	0	0	6
60	19	1	20	6	0	1	1	0	0	0	0	0	0	6
61	27	3	36	12	0	0	0	0	0	0	0	0	0	12

c) Those with the curves over P_∞:

No	n	k	deg	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
56	3	5	4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
57	5	1	6	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
58	13	3	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	17	5	18	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
60	19	1	20	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
61	27	3	36	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0

No	n	k	deg	42	43	44	45	46	47	48	49	50	51	52	53	54	55
56	3	5	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	5	1	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	13	3	14	1	0	0	0	0	0	0	0	0	0	0	0	1	0
59	17	5	18	0	0	0	0	0	0	0	0	0	0	0	0	0	1
60	19	1	20	0	0	0	0	0	0	0	0	0	0	0	0	0	1
61	27	3	36	1	0	0	0	0	0	0	0	0	0	0	0	1	1

d) Those of the Hecke curves with each other:

No	n	k	deg	56	57	58	59	60	61
56	3	5	4	-1	0	0	0	0	0
57	5	1	6	0	-2	0	0	0	0
58	13	3	14	0	0	-4	0	0	0
59	17	5	18	0	0	0	-2	0	0
60	19	1	20	0	0	0	0	-2	2
61	27	3	36	0	0	0	0	2	-4