

Invariants of the Surface $\tilde{Z}_{15,2}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	6	62	74	-48	76	8

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	L_∞	L	$2S_\infty$	$2S$
	1440	73	8	35	6	23	0	12	1	28	24	44	2	2

The Singularities of the associated singular surface $Z_{15,2}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 11]	+	2	1	[1, 11], [0, 1]	[122, 330, 225]	[17, 16, 17]
2	[1, 14]	+	1	2	[1, 14], [0, 1]	[197, 420, 225]	[2, 2, 113]
3	[4, 11]	+	1	3	[4, 11], [1, 3]	[137, 1110, 2250]	[2, 2, 113]
4	[4, 14]	+	2	1	[4, 29], [-1, -7]	[857, -6210, 11250]	[17, 16, 17]
5	[2, 12]	-	4	4	[2, 27], [-1, -13]	[733, -10590, 38250]	[13, -6, 18]
6	[3, 8]	-	4	4	[3, 8], [1, 3]	[73, 810, 2250]	[13, 6, 18]
7	[3, 13]	-	4	4	[3, 13], [-1, -4]	[178, -1650, 3825]	[13, 6, 18]
8	[7, 12]	-	4	4	[7, 12], [-3, -5]	[193, -2430, 7650]	[13, -6, 18]

The CM-Singularities above P_1 (those of type (-3))

- there are none of this type

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[1, 3]	-	6	1	[1, 3], [0, 1]	[13, 105, 225]	[13, 1, 13]
10	[1, 6]	-	6	1	[1, 6], [0, 1]	[43, 195, 225]	[7, 5, 25]
11	[1, 8]	-	6	1	[1, 8], [0, 1]	[73, 255, 225]	[7, -5, 25]
12	[2, 4]	-	6	1	[2, 19], [-1, -9]	[403, -5745, 20475]	[7, 5, 25]
13	[3, 1]	-	6	1	[3, 1], [-1, 0]	[13, -105, 225]	[13, -1, 13]
14	[4, 2]	-	6	1	[4, 17], [-1, -4]	[373, -2655, 4725]	[7, -5, 25]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[15, 2]	2	[8, 2]
16	[1, 3]	4	2	[3, 2]	2	[2, 2]
17	[1, 5]	2	3	[5, 2]	2	[3, 2]
18	[1, 6]	4	2	[3, 2]	2	[2, 2]
19	[1, 9]	4	2	[3, 2]	2	[2, 2]
20	[1, 10]	2	3	[5, 2]	2	[3, 2]
21	[1, 12]	4	2	[3, 2]	2	[2, 2]
22	[2, 0]	1	4	[15, 8]	2	[2, 8]
23	[2, 5]	2	5	[5, 3]	2	[2, 3]
24	[2, 10]	2	5	[5, 3]	2	[2, 3]
25	[4, 0]	1	6	[15, 2]	2	[8, 2]
26	[7, 0]	1	7	[15, 8]	2	[2, 8]

The Basic Curves on $\tilde{Z}_{15,2}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	35	35	0	-4
10	35	35	0	-4
11	23	23	0	-4
24	23	23	0	-4
25	1	1	0	-6
50	1	1	0	-6

The intersection matrix for the non-exceptional curves:

No	1	10	11	24	25	50
1	-4	356	0	240	0	48
10	356	-4	240	0	48	0
11	0	240	-4	158	0	32
24	240	0	158	-4	32	0
25	0	48	0	32	-6	4
50	48	0	32	0	4	-6

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	356
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	356	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...24)

No	11	12	13	14	15	16	17	18	19	20	21	22	23	24
11	-4	1	0	1	0	1	0	1	0	1	0	1	0	158
12	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
13	0	1	-2	0	0	0	0	0	0	0	0	0	0	1
14	1	0	0	-2	1	0	0	0	0	0	0	0	0	0
15	0	0	0	1	-2	0	0	0	0	0	0	0	0	1
16	1	0	0	0	0	-2	1	0	0	0	0	0	0	0
17	0	0	0	0	0	1	-2	0	0	0	0	0	0	1
18	1	0	0	0	0	0	0	-2	1	0	0	0	0	0
19	0	0	0	0	0	0	0	1	-2	0	0	0	0	1
20	1	0	0	0	0	0	0	0	0	-2	1	0	0	0
21	0	0	0	0	0	0	0	0	0	1	-2	0	0	1
22	1	0	0	0	0	0	0	0	0	0	0	-2	1	0
23	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
24	158	0	1	0	1	0	1	0	1	0	1	0	1	-4

The intersection matrix for the P_∞ -curves (curves 25...50)

No	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
25	-6	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
26	1	-8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0
34	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0
36	1	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0
38	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-8	0	0	0
42	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0
44	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	4	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

No	45	46	47	48	49	50
25	0	1	0	1	0	4
26	0	0	0	0	0	0
27	0	0	0	0	0	1
28	0	0	0	0	0	0
29	0	0	0	0	0	1
30	0	0	0	0	0	0
31	0	0	0	0	0	1
32	0	0	0	0	0	0
33	0	0	0	0	0	1
34	0	0	0	0	0	0
35	0	0	0	0	0	1
36	0	0	0	0	0	0
37	0	0	0	0	0	1
38	0	0	0	0	0	0
39	0	0	0	0	0	1
40	0	0	0	0	0	0
41	0	0	0	0	0	1
42	0	0	0	0	0	0
43	0	0	0	0	0	1
44	1	0	0	0	0	0
45	-3	0	0	0	0	1
46	0	-8	1	0	0	0
47	0	1	-2	0	0	1
48	0	0	0	-2	1	0
49	0	0	0	1	-8	1
50	1	0	1	0	1	-6

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{15,2}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
51	2	2	3	0	0	0	-1
52	2	7	3	0	0	0	-1
53	8	1	12	0	0	0	-2
54	8	4	12	0	0	0	-2
55	17	2	18	1	1	0	-2
56	17	7	18	1	1	0	-2
57	23	1	24	2	2	0	-2
58	23	4	24	2	2	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
51	2	2	3	1	0	0	1	0	0	0	0	0	1
52	2	7	3	1	0	1	0	0	0	0	0	0	1
53	8	1	12	6	0	0	0	0	0	0	0	0	6
54	8	4	12	6	0	0	0	0	0	0	0	0	6
55	17	2	18	8	1	0	0	1	0	0	0	0	8
56	17	7	18	8	1	0	0	1	0	0	0	0	8
57	23	1	24	12	0	0	0	0	0	0	0	0	12
58	23	4	24	12	0	0	0	0	0	0	0	0	12

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18	19	20	21	22	23	24
51	2	2	3	1	0	0	0	0	0	0	0	0	0	0	0	0	1
52	2	7	3	1	0	0	0	0	0	0	0	0	0	0	0	0	1
53	8	1	12	4	0	0	0	0	0	0	0	0	0	0	0	0	4
54	8	4	12	4	0	0	0	0	0	0	0	0	0	0	0	0	4
55	17	2	18	6	0	0	0	0	0	0	0	0	0	0	0	0	6
56	17	7	18	6	0	0	0	0	0	0	0	0	0	0	0	0	6
57	23	1	24	8	0	0	0	0	0	0	0	0	0	0	0	0	8
58	23	4	24	8	0	0	0	0	0	0	0	0	0	0	0	0	8

c) Those with the curves over P_∞ :

No	n	k	deg	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
51	2	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	2	7	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
53	8	1	12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
54	8	4	12	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
55	17	2	18	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	17	7	18	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
57	23	1	24	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
58	23	4	24	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

No	n	k	deg	45	46	47	48	49	50
51	2	2	3	0	1	0	0	1	0
52	2	7	3	0	0	0	0	0	0
53	8	1	12	0	1	0	0	1	0
54	8	4	12	0	0	1	1	0	0
55	17	2	18	0	1	0	0	1	1
56	17	7	18	0	0	0	0	0	1
57	23	1	24	0	0	0	0	0	1
58	23	4	24	0	0	1	1	0	1

d) Those of the Hecke curves with each other:

No	n	k	deg	51	52	53	54	55	56	57	58
51	2	2	3	-1	0	-2/15	2/15	0	0	2/15	-2/15
52	2	7	3	0	-1	2/15	-2/15	0	0	-2/15	2/15
53	8	1	12	-2/15	2/15	-2	0	-2/15	2/15	0	0
54	8	4	12	2/15	-2/15	0	-2	2/15	-2/15	0	0
55	17	2	18	0	0	-2/15	2/15	-2	0	2/15	-2/15
56	17	7	18	0	0	2/15	-2/15	0	-2	-2/15	2/15
57	23	1	24	2/15	-2/15	0	0	2/15	-2/15	-2	0
58	23	4	24	-2/15	2/15	0	0	-2/15	2/15	0	-2