

Invariants of the Surface $\tilde{Z}_{15,7}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	8	68	84	-50	86	22

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	1440	73	8	35	6	23	6	12	1	28	40	54	-3	-2

The Singularities of the associated singular surface $Z_{15,7}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 9]	+	2	1	[1, 9], [0, 1]	[82, 270, 225]	[13, 6, 18]
2	[4, 9]	+	2	2	[4, 9], [-1, -2]	[97, -660, 1125]	[13, -6, 18]
3	[6, 11]	+	2	2	[6, 11], [1, 2]	[157, 840, 1125]	[13, 6, 18]
4	[6, 14]	+	2	1	[6, 29], [1, 5]	[877, 4530, 5850]	[13, -6, 18]
5	[2, 8]	-	4	3	[2, 23], [-1, -11]	[533, -7650, 27450]	[17, -16, 17]
6	[2, 13]	-	4	3	[2, 13], [-1, -6]	[173, -2400, 8325]	[2, 2, 113]
7	[7, 8]	-	4	3	[7, 8], [-1, -1]	[113, -450, 450]	[2, 2, 113]
8	[7, 13]	-	4	3	[7, 13], [1, 2]	[218, 990, 1125]	[17, -16, 17]

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[1, 2]	+	2	1	[1, 2], [0, 1]	[7, 75, 225]	[7, 5, 25]
10	[2, 1]	+	2	1	[2, 1], [-1, 0]	[7, -75, 225]	[7, -5, 25]
11	[2, 6]	+	2	2	[2, 21], [-1, -10]	[487, -6975, 24975]	[13, 1, 13]
12	[2, 7]	+	2	2	[2, 7], [-1, -3]	[67, -885, 2925]	[13, -1, 13]
13	[3, 4]	+	2	3	[3, 4], [-1, -1]	[37, -315, 675]	[7, 5, 25]
14	[4, 3]	+	2	3	[4, 3], [1, 1]	[37, 315, 675]	[7, -5, 25]

The anti-CM-Singularities above P1 (those of type (-2, -2))

- there are none of this type

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[15, 7]	7	[3, 2, 2, 2, 2, 2, 2]
16	[1, 3]	4	2	[3, 1]	1	[3]
17	[1, 5]	2	3	[5, 2]	2	[3, 2]
18	[1, 6]	4	2	[3, 1]	1	[3]
19	[1, 9]	4	2	[3, 1]	1	[3]
20	[1, 10]	2	3	[5, 2]	2	[3, 2]
21	[1, 12]	4	2	[3, 1]	1	[3]
22	[2, 0]	1	4	[15, 13]	7	[2, 2, 2, 2, 2, 2, 3]
23	[2, 5]	2	5	[5, 3]	2	[2, 3]
24	[2, 10]	2	5	[5, 3]	2	[2, 3]
25	[4, 0]	1	6	[15, 7]	7	[3, 2, 2, 2, 2, 2, 2]
26	[7, 0]	1	7	[15, 13]	7	[2, 2, 2, 2, 2, 2, 3]

The Basic Curves on $\tilde{Z}_{15,7}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	35	35	0	-4
10	35	35	0	-4
11	23	23	0	-2
18	23	23	0	-2
19	1	1	0	-6
60	1	1	0	-6

The intersection matrix for the non-exceptional curves:

No	1	10	11	18	19	60
1	-4	356	0	240	0	48
10	356	-4	240	0	48	0
11	0	240	-2	158	0	32
18	240	0	158	-2	32	0
19	0	48	0	32	-6	4
60	48	0	32	0	4	-6

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	356
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	356	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...18)

No	11	12	13	14	15	16	17	18
11	-2	1	1	1	1	1	1	158
12	1	-3	0	0	0	0	0	1
13	1	0	-3	0	0	0	0	1
14	1	0	0	-3	0	0	0	1
15	1	0	0	0	-3	0	0	1
16	1	0	0	0	0	-3	0	1
17	1	0	0	0	0	0	-3	1
18	158	1	1	1	1	1	1	-2

The intersection matrix for the P_∞ -curves (curves 19...60)

No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
19	-6	1	0	0	0	0	0	0	1	1	0	1	1	1	0	1	1	0	0	0	0	0
20	1	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0
31	1	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0
32	1	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0
34	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0
35	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	4	0	0	0	0	0	0	1	1	0	1	1	1	0	1	1	0	0	0	0	0	0

No	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
19	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	4
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
44	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	1
53	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	1	1
60	1	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	-6	-6

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{15,7}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
61	7	2	8	0	0	0	-2
62	7	7	8	0	0	0	-2
63	13	1	14	0	0	0	-4
64	13	4	14	0	0	0	-4
65	22	2	36	2	2	0	-4
66	22	7	36	2	2	0	-4
67	28	1	48	2	2	0	-6
68	28	4	48	2	2	0	-6

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
61	7	2	8	4	0	0	0	0	0	0	0	0	4
62	7	7	8	4	0	0	0	0	0	0	0	0	4
63	13	1	14	6	1	0	0	1	0	0	0	0	6
64	13	4	14	6	0	1	1	0	0	0	0	0	6
65	22	2	36	18	0	0	0	0	0	0	0	0	18
66	22	7	36	18	0	0	0	0	0	0	0	0	18
67	28	1	48	24	0	0	0	0	0	0	0	0	24
68	28	4	48	24	0	0	0	0	0	0	0	0	24

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18
61	7	2	8	2	1	1	0	0	0	0	2
62	7	7	8	2	0	0	0	0	1	1	2
63	13	1	14	4	0	0	1	1	0	0	4
64	13	4	14	4	0	0	1	1	0	0	4
65	22	2	36	12	0	0	0	0	0	0	12
66	22	7	36	12	0	0	0	0	0	0	12
67	28	1	48	16	0	0	0	0	0	0	16
68	28	4	48	16	0	0	0	0	0	0	16

c) Those with the curves over P_∞ :

No	n	k	deg	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
61	7	2	8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	7	7	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	13	1	14	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
64	13	4	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
65	22	2	36	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
66	22	7	36	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
67	28	1	48	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
68	28	4	48	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

No	n	k	deg	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
61	7	2	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	7	7	8	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
63	13	1	14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
64	13	4	14	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
65	22	2	36	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
66	22	7	36	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
67	28	1	48	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
68	28	4	48	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0

d) Those of the Hecke curves with each other:

No	n	k	deg	61	62	63	64	65	66	67	68
61	7	2	8	-2	0	$-2/15$	$2/15$	0	0	$-8/15$	$8/15$
62	7	7	8	0	-2	$2/15$	$-2/15$	0	0	$8/15$	$-8/15$
63	13	1	14	$-2/15$	$2/15$	-4	0	$4/3$	$-4/3$	0	0
64	13	4	14	$2/15$	$-2/15$	0	-4	$-4/3$	$4/3$	0	0
65	22	2	36	0	0	$4/3$	$-4/3$	-4	2	$-2/3$	$2/3$
66	22	7	36	0	0	$-4/3$	$4/3$	2	-4	$2/3$	$-2/3$
67	28	1	48	$-8/15$	$8/15$	0	0	$-2/3$	$2/3$	-6	0
68	28	4	48	$8/15$	$-8/15$	0	0	$2/3$	$-2/3$	0	-6