Invariants of the Surface $\tilde{Z}_{5,1}$

Basic Numerical Invariants:

Geometric:

p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
0	12	12	-10	14	-2

Other:

m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
60	0	2	0	2	0	1	2	0	0	5	10	0	0

The Singularities of the associated singular surface $Z_{5,1}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	$[0,4]$	$+$	1	1	$[5,4],[1,1]$	$[41,90,50]$	$[1,0,25]$
2	$[0,3]$	$-$	1	2	$[5,3],[-2,-1]$	$[34,-130,125]$	$[1,0,25]$

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
3	$[0,1]$	$+$	1	1	$[0,1],[-1,0]$	$[1,-5,25]$	$[1,1,19]$

The anti-CM-Singularities above P1 (those of type $(-2,-2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
4	$[0,2]$	$-$	1	2	$[5,2],[2,1]$	$[39,165,175]$	$[1,1,19]$

The Singularities above P_∞:

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
5	$[1,0]$	1	1	$[5,1]$	1	$[5]$
6	$[2,0]$	1	2	$[5,4]$	4	$[2,2,2,2]$

The Basic Curves on $\tilde{Z}_{5,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	0	0	0	-1
4	0	0	0	-1
5	0	0	0	-1
9	0	0	0	-1
10	0	0	0	-1
16	0	0	0	-1

The intersection matrix for the non-exceptional curves:

No	1	4	5	9	10	16
1	-1	14	0	10	0	6
4	14	-1	10	0	6	0
5	0	10	-1	6	0	4
9	10	0	6	-1	4	0
10	0	6	0	4	-1	2
16	6	0	4	0	2	-1

The intersection matrix for the P_0-curves (curves 1…4)

No	1	2	3	4
1	-1	1	1	14
2	1	-2	0	1
3	1	0	-2	1
4	14	1	1	-1

The intersection matrix for the P_1-curves (curves 5…9)

No	5	6	7	8	9
5	-1	1	1	0	6
6	1	-3	0	0	1
7	1	0	-2	1	0
8	0	0	1	-2	1
9	6	1	0	1	-1

The intersection matrix for the P_∞-curves (curves 10…16)

No	10	11	12	13	14	15	16
10	-1	1	1	0	0	0	2
11	1	-5	0	0	0	0	1
12	1	0	-2	1	0	0	0
13	0	0	1	-2	1	0	0
14	0	0	0	1	-2	1	0
15	0	0	0	0	1	-2	1
16	2	1	0	0	0	1	-1

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{5,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
17	1	1	1	0	0	0	-1
18	4	2	6	0	0	0	-1
19	6	1	12	0	0	0	0
20	9	2	12	1	0	1	2
21	11	1	12	2	1	1	4
22	14	2	24	6	1	5	14
23	16	1	24	6	0	6	14
24	19	2	20	5	1	4	10
25	21	1	32	10	1	9	22
26	24	2	48	24	1	23	54
27	26	1	42	23	2	21	48
28	29	2	30	13	2	11	26

Their intersection numbers with other curves:

a) Those with the curves over P_0:

No	n	k	deg	1	2	3	4
17	1	1	1	0	1	0	0
18	4	2	6	3	0	0	3
19	6	1	12	6	0	0	6
20	9	2	12	6	0	0	6
21	11	1	12	6	0	0	6
22	14	2	24	12	0	0	12
23	16	1	24	12	0	0	12
24	19	2	20	10	0	0	10
25	21	1	32	16	0	0	16
26	24	2	48	24	0	0	24
27	26	1	42	20	2	0	20
28	29	2	30	14	2	0	14

b) Those with the curves over P_1:

No	n	k	deg	5	6	7	8	9
17	1	1	1	0	1	0	0	0
18	4	2	6	2	0	0	0	2
19	6	1	12	4	0	0	0	4
20	9	2	12	4	0	0	0	4
21	11	1	12	4	0	0	0	4
22	14	2	24	8	0	0	0	8
23	16	1	24	8	0	0	0	8
24	19	2	20	6	2	0	0	6
25	21	1	32	10	2	0	0	10
26	24	2	48	16	0	0	0	16
27	26	1	42	14	0	0	0	14
28	29	2	30	10	0	0	0	10

c) Those with the curves over P_∞:

No	n	k	deg	10	11	12	13	14	15	16
17	1	1	1	0	1	0	0	0	0	0
18	4	2	6	0	1	1	0	0	1	0
19	6	1	12	1	2	0	1	1	0	1
20	9	2	12	1	2	1	0	0	1	1
21	11	1	12	2	2	0	0	0	0	2
22	14	2	24	3	4	1	0	0	1	3
23	16	1	24	3	4	1	0	0	1	3
24	19	2	20	3	0	1	0	0	1	3
25	21	1	32	4	2	1	1	1	1	4
26	24	2	48	6	8	1	1	1	1	6
27	26	1	42	6	2	2	0	0	2	6
28	29	2	30	5	0	1	0	0	1	5

d) Those of the Hecke curves with each other:

No	n	k	deg	17	18	19	20	21	22	23	24	25	26	27	28
17	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
18	4	2	6	0	-1	0	0	2	2	2	2	2	4	4	4
19	6	1	12	0	0	0	2	4	6	6	6	6	10	12	10
20	9	2	12	0	0	2	2	4	6	6	6	8	12	12	10
21	11	1	12	0	2	4	4	4	8	8	8	12	16	16	12
22	14	2	24	0	2	6	6	8	14	14	14	20	28	28	22
23	16	1	24	0	2	6	6	8	14	14	14	20	28	28	22
24	19	2	20	0	2	6	6	8	14	14	10	16	28	24	18
25	21	1	32	0	2	6	8	12	20	20	16	22	38	36	28
26	24	2	48	0	4	10	12	16	28	28	28	38	54	56	44
27	26	1	42	0	4	12	12	16	28	28	24	36	56	48	36
28	29	2	30	0	4	10	10	12	22	22	18	28	44	36	26