

Invariants of the Surface $\tilde{Z}_{5,2}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	0	11	11	-9	13	-1

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	L_∞	L	$2S_\infty$	$2S$
	60	0	2	0	2	0	1	2	0	0	4	9	0	0

The Singularities of the associated singular surface $Z_{5,2}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 4]	+	1	1	[1, 4], [0, 1]	[17, 40, 25]	[2, 2, 13]
2	[2, 3]	-	1	2	[2, 3], [-1, -1]	[13, -50, 50]	[2, 2, 13]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
3	[1, 2]	+	1	1	[1, 2], [0, 1]	[7, 25, 25]	[3, 3, 7]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
4	[1, 1]	-	1	2	[1, 1], [-1, 0]	[3, -15, 25]	[3, 3, 7]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
5	[1, 0]	1	1	[5, 2]	2	[3, 2]
6	[2, 0]	1	2	[5, 3]	2	[2, 3]

The Basic Curves on $\tilde{Z}_{5,2}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	0	0	0	-1
4	0	0	0	-1
5	0	0	0	-1
9	0	0	0	-1
10	0	0	0	-1
15	0	0	0	-1

The intersection matrix for the non-exceptional curves:

No	1	4	5	9	10	15
1	-1	14	0	10	0	6
4	14	-1	10	0	6	0
5	0	10	-1	6	0	4
9	10	0	6	-1	4	0
10	0	6	0	4	-1	2
15	6	0	4	0	2	-1

The intersection matrix for the P_0 -curves (curves 1...4)

No	1	2	3	4
1	-1	1	1	14
2	1	-2	0	1
3	1	0	-2	1
4	14	1	1	-1

The intersection matrix for the P_1 -curves (curves 5...9)

No	5	6	7	8	9
5	-1	1	1	0	6
6	1	-3	0	0	1
7	1	0	-2	1	0
8	0	0	1	-2	1
9	6	1	0	1	-1

The intersection matrix for the P_∞ -curves (curves 10...15)

No	10	11	12	13	14	15
10	-1	1	0	1	0	2
11	1	-3	1	0	0	0
12	0	1	-2	0	0	1
13	1	0	0	-2	1	0
14	0	0	0	1	-3	1
15	2	0	1	0	1	-1

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{5,2}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
16	2	2	3	0	0	0	-1
17	3	1	4	0	0	0	-1
18	7	2	8	0	0	0	0
19	8	1	12	1	0	1	2
20	12	2	24	3	0	3	8
21	13	1	14	1	0	1	2
22	17	2	18	4	1	3	8
23	18	1	36	11	0	11	26
24	22	2	36	17	2	15	36
25	23	1	24	9	2	7	18
26	27	2	36	17	1	16	36
27	28	1	48	27	2	25	58

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4
16	2	2	3	1	1	0	1
17	3	1	4	2	0	0	2
18	7	2	8	4	0	0	4
19	8	1	12	6	0	0	6
20	12	2	24	12	0	0	12
21	13	1	14	6	2	0	6
22	17	2	18	8	2	0	8
23	18	1	36	18	0	0	18
24	22	2	36	18	0	0	18
25	23	1	24	12	0	0	12
26	27	2	36	18	0	0	18
27	28	1	48	24	0	0	24

b) Those with the curves over P_1 :

No	n	k	deg	5	6	7	8	9
16	2	2	3	1	0	0	0	1
17	3	1	4	1	1	0	0	1
18	7	2	8	2	2	0	0	2
19	8	1	12	4	0	0	0	4
20	12	2	24	8	0	0	0	8
21	13	1	14	4	2	0	0	4
22	17	2	18	6	0	0	0	6
23	18	1	36	12	0	0	0	12
24	22	2	36	12	0	0	0	12
25	23	1	24	8	0	0	0	8
26	27	2	36	12	0	0	0	12
27	28	1	48	16	0	0	0	16

c) Those with the curves over P_∞ :

No	n	k	deg	10	11	12	13	14	15
16	2	2	3	0	1	0	0	1	0
17	3	1	4	0	0	1	1	0	0
18	7	2	8	1	1	0	0	1	1
19	8	1	12	1	1	1	1	1	1
20	12	2	24	2	2	2	2	2	2
21	13	1	14	2	0	1	1	0	2
22	17	2	18	3	1	0	0	1	3
23	18	1	36	4	4	1	1	4	4
24	22	2	36	5	1	2	2	1	5
25	23	1	24	4	0	1	1	0	4
26	27	2	36	5	1	2	2	1	5
27	28	1	48	6	2	3	3	2	6

d) Those of the Hecke curves with each other:

No	n	k	deg	16	17	18	19	20	21	22	23	24	25	26	27
16	2	2	3	-1	0	0	0	0	0	0	0	2	2	2	2
17	3	1	4	0	-1	0	0	0	0	2	2	2	2	2	2
18	7	2	8	0	0	0	2	4	2	4	6	8	6	8	10
19	8	1	12	0	0	2	2	4	4	6	8	10	8	10	12
20	12	2	24	0	0	4	4	8	8	12	16	20	16	20	24
21	13	1	14	0	0	2	4	8	2	6	14	14	10	14	18
22	17	2	18	0	2	4	6	12	6	8	18	20	14	20	26
23	18	1	36	0	2	6	8	16	14	18	26	34	26	34	42
24	22	2	36	2	2	8	10	20	14	20	34	36	26	36	46
25	23	1	24	2	2	6	8	16	10	14	26	26	18	26	34
26	27	2	36	2	2	8	10	20	14	20	34	36	26	36	46
27	28	1	48	2	2	10	12	24	18	26	42	46	34	46	58