

Invariants of the Surface $\tilde{Z}_{6,1}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	0	14	14	-12	16	-4

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	72	1	4	0	3	0	3	3	0	2	3	10	0	1

The Singularities of the associated singular surface $Z_{6,1}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 5]	+	1	1	[6, 5], [1, 1]	[61, 132, 72]	[1, 0, 36]
2	[3, 4]	+	1	2	[3, 4], [-1, -1]	[25, -84, 72]	[4, 0, 9]
3	[1, 4]	-	2	3	[1, 4], [0, 1]	[17, 48, 36]	[5, -4, 8]
4	[2, 5]	-	2	3	[2, 5], [-1, -2]	[29, -144, 180]	[5, 4, 8]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -6, 36]	[1, 0, 27]
6	[1, 2]	+	2	2	[1, 2], [0, 1]	[7, 30, 36]	[4, -2, 7]
7	[2, 1]	+	2	2	[2, 1], [-1, 0]	[7, -30, 36]	[4, 2, 7]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

- there are none of this type

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
8	[1, 0]	1	1	[6, 1]	1	[6]
9	[1, 2]	1	2	[2, 1]	1	[2]
10	[1, 3]	1	3	[3, 1]	1	[3]

The Basic Curves on $\tilde{Z}_{6,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	0	0	0	-2
6	0	0	0	-2
7	0	0	0	-1
11	0	0	0	-1
12	0	0	0	-1
16	0	0	0	-1

The intersection matrix for the non-exceptional curves:

No	1	6	7	11	12	16
1	-2	16	0	12	0	6
6	16	-2	12	0	6	0
7	0	12	-1	7	0	4
11	12	0	7	-1	4	0
12	0	6	0	4	-1	1
16	6	0	4	0	1	-1

The intersection matrix for the P_0 -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	16
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	16	1	1	1	1	-2

The intersection matrix for the P_1 -curves (curves 7...11)

No	7	8	9	10	11
7	-1	1	1	1	7
8	1	-3	0	0	1
9	1	0	-3	0	1
10	1	0	0	-3	1
11	7	1	1	1	-1

The intersection matrix for the P_∞ -curves (curves 12...16)

No	12	13	14	15	16
12	-1	1	1	1	1
13	1	-6	0	0	1
14	1	0	-2	0	1
15	1	0	0	-3	1
16	1	1	1	1	-1

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{6,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
17	1	1	1	0	0	0	-1
18	7	1	8	0	0	0	0
19	13	1	14	1	0	1	2
20	19	1	20	4	1	3	8
21	25	1	30	7	0	7	16

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6
17	1	1	1	0	1	0	0	0	0
18	7	1	8	4	0	0	0	0	4
19	13	1	14	6	0	2	0	0	6
20	19	1	20	10	0	0	0	0	10
21	25	1	30	14	0	2	0	0	14

b) Those with the curves over P_1 :

No	n	k	deg	7	8	9	10	11
17	1	1	1	0	1	0	0	0
18	7	1	8	2	0	1	1	2
19	13	1	14	4	0	1	1	4
20	19	1	20	6	0	1	1	6
21	25	1	30	10	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	12	13	14	15	16
17	1	1	1	0	1	0	0	0
18	7	1	8	1	2	0	0	1
19	13	1	14	2	2	0	0	2
20	19	1	20	3	2	0	0	3
21	25	1	30	4	6	0	0	4

d) Those of the Hecke curves with each other:

No	n	k	deg	17	18	19	20	21
17	1	1	1	-1	0	0	0	0
18	7	1	8	0	0	2	4	4
19	13	1	14	0	2	2	6	8
20	19	1	20	0	4	6	8	16
21	25	1	30	0	4	8	16	16