

Invariants of the Surface $\tilde{Z}_{6,5}$

Basic Numerical Invariants:

Geometric:

p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
1	20	22	-16	24	0

Other:

m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2\mathbb{S}_\infty$	$2\mathbb{S}$
72	1	4	0	3	0	0	3	0	2	8	18	-1	-1

The Singularities of the associated singular surface $Z_{6,5}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 4]	+	2	1	[1, 4], [0, 1]	[17, 48, 36]	[5, -4, 8]
2	[2, 5]	+	2	1	[2, 5], [-1, -2]	[29, -144, 180]	[5, 4, 8]
3	[0, 5]	-	2	2	[6, 5], [1, 1]	[61, 132, 72]	[1, 0, 36]
4	[3, 4]	-	2	2	[3, 4], [-1, -1]	[25, -84, 72]	[4, 0, 9]

The CM-Singularities above P_1 (those of type (-3))

– there are none of this type

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	[0, 1]	-	3	1	[0, 1], [-1, 0]	[1, -6, 36]	[1, 0, 27]
6	[1, 2]	-	3	1	[1, 2], [0, 1]	[7, 30, 36]	[4, -2, 7]
7	[2, 1]	-	3	1	[2, 1], [-1, 0]	[7, -30, 36]	[4, 2, 7]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
8	[1, 0]	1	1	[6, 5]	5	[2, 2, 2, 2, 2]
9	[1, 2]	1	2	[2, 1]	1	[2]
10	[1, 3]	1	3	[3, 2]	2	[2, 2]

The Basic Curves on $\tilde{Z}_{6,5}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	0	0	0	-2
6	0	0	0	-2
7	0	0	0	-2
14	0	0	0	-2
15	0	0	0	-2
24	0	0	0	-2

The intersection matrix for the non-exceptional curves:

No	1	6	7	14	15	24
1	-2	16	0	12	0	6
6	16	-2	12	0	6	0
7	0	12	-2	7	0	4
14	12	0	7	-2	4	0
15	0	6	0	4	-2	1
24	6	0	4	0	1	-2

The intersection matrix for the P_0 -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	16
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	16	1	1	1	1	-2

The intersection matrix for the P_1 -curves (curves 7...14)

No	7	8	9	10	11	12	13	14
7	-2	1	0	1	0	1	0	7
8	1	-2	1	0	0	0	0	0
9	0	1	-2	0	0	0	0	1
10	1	0	0	-2	1	0	0	0
11	0	0	0	1	-2	0	0	1
12	1	0	0	0	0	-2	1	0
13	0	0	0	0	0	1	-2	1
14	7	0	1	0	1	0	1	-2

The intersection matrix for the P_∞ -curves (curves 15...24)

No	15	16	17	18	19	20	21	22	23	24
15	-2	1	0	0	0	0	1	1	0	1
16	1	-2	1	0	0	0	0	0	0	0
17	0	1	-2	1	0	0	0	0	0	0
18	0	0	1	-2	1	0	0	0	0	0
19	0	0	0	1	-2	1	0	0	0	0
20	0	0	0	0	1	-2	0	0	0	1
21	1	0	0	0	0	0	-2	0	0	1
22	1	0	0	0	0	0	0	-2	1	0
23	0	0	0	0	0	0	0	1	-2	1
24	1	0	0	0	0	1	1	0	1	-2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{6,5}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
25	5	1	6	0	0	0	-2
26	11	1	12	2	1	1	2
27	17	1	18	4	1	3	6
28	23	1	24	8	2	6	14
29	29	1	30	12	2	10	22

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6
25	5	1	6	2	1	1	0	0	2
26	11	1	12	6	0	0	0	0	6
27	17	1	18	8	1	1	0	0	8
28	23	1	24	12	0	0	0	0	12
29	29	1	30	14	1	1	0	0	14

b) Those with the curves over P_1 :

No	n	k	deg	7	8	9	10	11	12	13	14
25	5	1	6	2	0	0	0	0	0	0	2
26	11	1	12	4	0	0	0	0	0	0	4
27	17	1	18	6	0	0	0	0	0	0	6
28	23	1	24	8	0	0	0	0	0	0	8
29	29	1	30	10	0	0	0	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	15	16	17	18	19	20	21	22	23	24
25	5	1	6	0	1	0	0	0	1	0	0	0	0
26	11	1	12	1	1	0	0	0	1	0	0	0	1
27	17	1	18	2	1	0	0	0	1	0	0	0	2
28	23	1	24	3	1	0	0	0	1	0	0	0	3
29	29	1	30	4	1	0	0	0	1	0	0	0	4

d) Those of the Hecke curves with each other:

No	n	k	deg	25	26	27	28	29
25	5	1	6	-2	0	0	2	2
26	11	1	12	0	2	4	6	8
27	17	1	18	0	4	6	10	12
28	23	1	24	2	6	10	14	18
29	29	1	30	2	8	12	18	22