

# Invariants of the Surface $\tilde{Z}_{7,1}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	0	16	16	-14	18	-6

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	168	3	4	1	2	1	1	3	0	2	5	12	1	1

## The Singularities of the associated singular surface $Z_{7,1}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[0, 6]	+	1	1	[7, 6], [1, 1]	[85, 182, 98]	[1, 0, 49]
2	[2, 5]	+	1	2	[2, 5], [-1, -2]	[29, -168, 245]	[2, 2, 25]
3	[2, 4]	-	2	3	[2, 11], [-1, -5]	[125, -798, 1274]	[5, -2, 10]
4	[3, 5]	-	2	3	[3, 5], [1, 2]	[34, 182, 245]	[5, 2, 10]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
5	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -7, 49]	[1, 1, 37]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
6	[1, 3]	-	1	2	[1, 3], [0, 1]	[13, 49, 49]	[3, 3, 13]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
7	[1, 0]	1	1	[7, 1]	1	[7]
8	[2, 0]	1	2	[7, 2]	2	[4, 2]
9	[3, 0]	1	3	[7, 4]	2	[2, 4]

## The Basic Curves on $\tilde{Z}_{7,1}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	1	1	0	-2
6	1	1	0	-2
7	1	1	0	-1
11	1	1	0	-1
12	0	0	0	-1
18	0	0	0	-1

### The intersection matrix for the non-exceptional curves:

No	1	6	7	11	12	18
1	-2	40	0	28	0	12
6	40	-2	28	0	12	0
7	0	28	-1	18	0	8
11	28	0	18	-1	8	0
12	0	12	0	8	-1	3
18	12	0	8	0	3	-1

The intersection matrix for the  $P_0$ -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	40
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	40	1	1	1	1	-2

The intersection matrix for the  $P_1$ -curves (curves 7...11)

No	7	8	9	10	11
7	-1	1	1	0	18
8	1	-3	0	0	1
9	1	0	-2	1	0
10	0	0	1	-2	1
11	18	1	0	1	-1

The intersection matrix for the  $P_\infty$ -curves (curves 12...18)

No	12	13	14	15	16	17	18
12	-1	1	1	0	1	0	3
13	1	-7	0	0	0	0	1
14	1	0	-4	1	0	0	0
15	0	0	1	-2	0	0	1
16	1	0	0	0	-2	1	0
17	0	0	0	0	1	-4	1
18	3	1	0	1	0	1	-1

The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{7,1}$  for  $n \leq 30$

Their basic properties:

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
19	1	1	1	0	0	0	-1
20	2	2	3	0	0	0	-1
21	4	3	6	0	0	0	-1
22	8	1	12	0	0	0	0
23	9	2	12	0	0	0	0
24	11	3	12	1	1	0	0
25	15	1	24	2	1	1	4
26	16	2	24	2	0	2	4
27	18	3	36	3	0	3	8
28	22	1	36	6	2	4	12
29	23	2	24	3	2	1	4
30	25	3	30	2	0	2	4
31	29	1	30	5	2	3	8
32	30	2	72	16	3	13	36

Their intersection numbers with other curves:

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6
19	1	1	1	0	1	0	0	0	0
20	2	2	3	1	0	1	0	0	1
21	4	3	6	3	0	0	0	0	3
22	8	1	12	6	0	0	0	0	6
23	9	2	12	6	0	0	0	0	6
24	11	3	12	6	0	0	0	0	6
25	15	1	24	12	0	0	0	0	12
26	16	2	24	12	0	0	0	0	12
27	18	3	36	18	0	0	0	0	18
28	22	1	36	18	0	0	0	0	18
29	23	2	24	12	0	0	0	0	12
30	25	3	30	14	0	2	0	0	14
31	29	1	30	14	0	2	0	0	14
32	30	2	72	36	0	0	0	0	36

b) Those with the curves over  $P_1$ :

No	$n$	$k$	deg	7	8	9	10	11
19	1	1	1	0	1	0	0	0
20	2	2	3	1	0	0	0	1
21	4	3	6	2	0	0	0	2
22	8	1	12	4	0	0	0	4
23	9	2	12	4	0	0	0	4
24	11	3	12	4	0	0	0	4
25	15	1	24	8	0	0	0	8
26	16	2	24	8	0	0	0	8
27	18	3	36	12	0	0	0	12
28	22	1	36	12	0	0	0	12
29	23	2	24	8	0	0	0	8
30	25	3	30	10	0	0	0	10
31	29	1	30	10	0	0	0	10
32	30	2	72	24	0	0	0	24

c) Those with the curves over  $P_\infty$ :

No	$n$	$k$	deg	12	13	14	15	16	17	18
19	1	1	1	0	1	0	0	0	0	0
20	2	2	3	0	0	1	0	0	1	0
21	4	3	6	0	1	0	1	1	0	0
22	8	1	12	1	2	1	0	0	1	1
23	9	2	12	1	2	1	0	0	1	1
24	11	3	12	1	0	0	1	1	0	1
25	15	1	24	2	2	1	1	1	1	2
26	16	2	24	2	2	1	1	1	1	2
27	18	3	36	3	4	2	1	1	2	3
28	22	1	36	4	2	2	0	0	2	4
29	23	2	24	3	0	1	0	0	1	3
30	25	3	30	3	4	0	1	1	0	3
31	29	1	30	4	2	0	0	0	0	4
32	30	2	72	6	6	3	3	3	3	6

d) Those of the Hecke curves with each other:

No	$n$	$k$	deg	19	20	21	22	23	24	25	26	27	28	29	30	31	32
19	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	2	2	3	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
21	4	3	6	0	0	-1	0	0	0	0	0	0	2	2	0	2	0
22	8	1	12	0	0	0	0	0	2	2	2	2	4	4	2	4	6
23	9	2	12	0	0	0	0	0	2	2	2	2	4	4	2	4	6
24	11	3	12	0	0	0	2	2	0	2	2	4	4	2	4	4	6
25	15	1	24	0	0	0	2	2	2	4	4	6	8	6	6	8	12
26	16	2	24	0	0	0	2	2	2	4	4	6	8	6	6	8	12
27	18	3	36	0	0	0	2	2	4	6	6	8	12	10	8	12	18
28	22	1	36	0	0	2	4	4	4	8	8	12	12	8	12	12	24
29	23	2	24	0	0	2	4	4	2	6	6	10	8	4	10	8	18
30	25	3	30	0	0	0	2	2	4	6	6	8	12	10	4	8	18
31	29	1	30	0	0	2	4	4	4	8	8	12	12	8	8	8	24
32	30	2	72	0	0	0	6	6	6	12	12	18	24	18	18	24	36