Invariants of the Surface $\tilde{Z}_{7,3}$

Basic Numerical Invariants:

Geometric:

p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
1	21	23	-17	25	-1

Other:

m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
168	3	4	1	2	1	1	3	0	2	12	19	-1	-1

The Singularities of the associated singular surface $Z_{7,3}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	$[1,4]$	$+$	2	1	$[1,4],[0,1]$	$[17,56,49]$	$[5,2,10]$
2	$[3,6]$	$+$	2	1	$[3,13],[-1,-4]$	$[178,-770,833]$	$[5,-2,10]$
3	$[0,5]$	$-$	2	2	$[7,5],[-3,-2]$	$[74,-434,637]$	$[1,0,49]$
4	$[3,4]$	$-$	2	2	$[3,4],[-1,-1]$	$[25,-98,98]$	$[2,2,25]$

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	$[1,1]$	$+$	1	1	$[1,1],[-1,0]$	$[3,-21,49]$	$[3,3,13]$

The anti-CM-Singularities above P1 (those of type $(-2,-2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
6	$[0,2]$	$-$	1	2	$[7,2],[3,1]$	$[67,413,637]$	$[1,1,37]$

The Singularities above P_∞:

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
7	$[1,0]$	1	1	$[7,3]$	3	$[3,2,2]$
8	$[2,0]$	1	2	$[7,6]$	6	$[2,2,2,2,2,2]$
9	$[3,0]$	1	3	$[7,5]$	3	$[2,2,3]$

The Basic Curves on $\tilde{Z}_{7,3}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	1	1	0	-2
6	1	1	0	-2
7	1	1	0	-1
11	1	1	0	-1
12	0	0	0	-2
25	0	0	0	-2

The intersection matrix for the non-exceptional curves:

No	1	6	7	11	12	25
1	-2	40	0	28	0	12
6	40	-2	28	0	12	0
7	0	28	-1	18	0	8
11	28	0	18	-1	8	0
12	0	12	0	8	-2	3
25	12	0	8	0	3	-2

The intersection matrix for the P_0-curves (curves 1...6)

No	1	2	3	4	5	6
1	−2	1	1	1	1	40
2	1	−2	0	0	0	1
3	1	0	−2	0	0	1
4	1	0	0	−2	0	1
5	1	0	0	0	−2	1
6	40	1	1	1	1	−2

The intersection matrix for the P_1-curves (curves 7...11)

No	7	8	9	10	11
7	−1	1	1	0	18
8	1	−3	0	0	1
9	1	0	−2	1	0
10	0	0	1	−2	1
11	18	1	0	1	−1

The intersection matrix for the P_∞-curves (curves 12...25)

No	12	13	14	15	16	17	18	19	20	21	22	23	24	25
12	−2	1	0	0	1	0	0	0	0	0	1	0	0	3
13	1	−3	1	0	0	0	0	0	0	0	0	0	0	0
14	0	1	−2	1	0	0	0	0	0	0	0	0	0	0
15	0	0	1	−2	0	0	0	0	0	0	0	0	0	1
16	1	0	0	0	−2	1	0	0	0	0	0	0	0	0
17	0	0	0	0	1	−2	1	0	0	0	0	0	0	0
18	0	0	0	0	0	1	−2	1	0	0	0	0	0	0
19	0	0	0	0	0	0	1	−2	1	0	0	0	0	0
20	0	0	0	0	0	0	0	1	−2	1	0	0	0	0
21	0	0	0	0	0	0	0	0	1	−2	0	0	0	1
22	1	0	0	0	0	0	0	0	0	0	−2	1	0	0
23	0	0	0	0	0	0	0	0	0	0	1	−2	1	0
24	0	0	0	0	0	0	0	0	0	0	0	1	−3	1
25	3	0	0	1	0	0	0	0	0	1	0	0	1	−2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{7,3}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
26	3	2	4	0	0	0	−1
27	5	1	6	0	0	0	−2
28	6	3	12	0	0	0	−2
29	10	2	18	0	0	0	−2
30	12	1	24	0	0	0	−2
31	13	3	14	0	0	0	−2
32	17	2	18	2	1	1	2
33	19	1	20	2	1	1	2
34	20	3	36	4	1	3	6
35	24	2	48	6	1	5	10
36	26	1	42	8	2	6	14
37	27	3	36	6	1	5	10

Their intersection numbers with other curves:

a) Those with the curves over P_0:

No	n	k	deg	1	2	3	4	5	6
26	3	2	4	2	0	0	0	0	2
27	5	1	6	2	1	1	0	0	2
28	6	3	12	6	0	0	0	0	6
29	10	2	18	8	1	1	0	0	8
30	12	1	24	12	0	0	0	0	12
31	13	3	14	6	1	1	0	0	6
32	17	2	18	8	1	1	0	0	8
33	19	1	20	10	0	0	0	0	10
34	20	3	36	18	0	0	0	0	18
35	24	2	48	24	0	0	0	0	24
36	26	1	42	20	1	1	0	0	20
37	27	3	36	18	0	0	0	0	18

b) Those with the curves over P_1:

No	n	k	deg	7	8	9	10	11
26	3	2	4	1	1	0	0	1
27	5	1	6	2	0	0	0	2
28	6	3	12	4	0	0	0	4
29	10	2	18	6	0	0	0	6
30	12	1	24	8	0	0	0	8
31	13	3	14	4	2	0	0	4
32	17	2	18	6	0	0	0	6
33	19	1	20	6	2	0	0	6
34	20	3	36	12	0	0	0	12
35	24	2	48	16	0	0	0	16
36	26	1	42	14	0	0	0	14
37	27	3	36	12	0	0	0	12

c) Those with the curves over P_∞:

No	n	k	deg	12	13	14	15	16	17	18	19	20	21	22	23	24	25
26	3	2	4	0	1	0	0	0	0	0	0	0	0	0	0	1	0
27	5	1	6	0	0	0	1	0	0	0	0	0	0	1	0	0	0
28	6	3	12	0	0	1	0	1	0	0	0	0	1	0	1	0	0
29	10	2	18	1	1	0	0	0	1	0	0	1	0	0	0	1	1
30	12	1	24	1	1	0	1	0	0	1	1	0	0	1	0	1	1
31	13	3	14	1	0	0	0	1	0	0	0	0	1	0	0	0	1
32	17	2	18	2	1	0	0	0	0	0	0	0	0	0	0	1	2
33	19	1	20	2	0	0	1	0	0	0	0	0	0	1	0	0	2
34	20	3	36	2	1	1	1	1	0	0	0	0	1	1	1	1	2
35	24	2	48	3	1	2	1	1	0	0	0	0	1	1	2	1	3
36	26	1	42	4	2	0	1	0	0	0	0	0	0	1	0	2	4
37	27	3	36	3	2	0	0	1	0	0	0	0	1	0	0	2	3

d) Those of the Hecke curves with each other:

No	n	k	deg	26	27	28	29	30	31	32	33	34	35	36	37
26	3	2	4	-1	0	0	0	0	0	0	0	0	0	0	0
27	5	1	6	0	-2	0	0	0	0	0	0	0	0	0	2
28	6	3	12	0	0	-2	0	0	0	2	2	0	0	4	2
29	10	2	18	0	0	0	-2	0	0	2	4	4	6	6	4
30	12	1	24	0	0	0	0	-2	2	4	4	4	6	8	6
31	13	3	14	0	0	0	0	2	-2	2	2	4	6	6	4
32	17	2	18	0	0	2	2	4	2	2	4	6	8	6	6
33	19	1	20	0	0	2	4	4	2	4	2	6	8	8	8
34	20	3	36	0	0	0	4	4	4	6	6	6	8	12	10
35	24	2	48	0	0	0	6	6	6	8	8	8	10	16	14
36	26	1	42	0	0	4	6	8	6	6	8	12	16	14	14
37	27	3	36	0	2	2	4	6	4	6	8	10	14	14	10