

# Invariants of the Surface $\tilde{Z}_{8,1}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	0	20	20	-18	22	-10

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$\mathbb{L}_\infty$	$\mathbb{L}$	$2S_\infty$	$2S$
	192	5	4	2	4	1	2	4	0	4	4	14	2	2

## The Singularities of the associated singular surface $Z_{8,1}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[0, 5]	+	1	1	[8, 5], [3, 2]	[89, 544, 832]	[1, 0, 64]
2	[0, 7]	+	1	2	[8, 7], [1, 1]	[113, 240, 128]	[1, 0, 64]
3	[4, 5]	+	1	3	[4, 5], [-1, -1]	[41, -144, 128]	[4, 4, 17]
4	[4, 7]	+	1	4	[4, 7], [1, 2]	[65, 288, 320]	[4, 4, 17]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
5	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -8, 64]	[1, 0, 48]
6	[0, 3]	+	1	2	[8, 3], [-3, -1]	[97, -568, 832]	[1, 0, 48]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
7	[1, 2]	-	2	3	[1, 2], [0, 1]	[7, 40, 64]	[7, -2, 7]
8	[2, 1]	-	2	3	[2, 1], [-1, 0]	[7, -40, 64]	[7, 2, 7]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
9	[1, 0]	1	1	[8, 1]	1	[8]
10	[1, 2]	1	2	[2, 1]	1	[2]
11	[1, 4]	1	3	[4, 1]	1	[4]
12	[3, 0]	1	4	[8, 1]	1	[8]

## The Basic Curves on $\tilde{Z}_{8,1}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	2	2	0	-2
6	2	2	0	-2
7	1	1	0	-2
14	1	1	0	-2
15	0	0	0	-1
20	0	0	0	-1

### The intersection matrix for the non-exceptional curves:

No	1	6	7	14	15	20
1	-2	46	0	32	0	12
6	46	-2	32	0	12	0
7	0	32	-2	20	0	8
14	32	0	20	-2	8	0
15	0	12	0	8	-1	2
20	12	0	8	0	2	-1

The intersection matrix for the  $P_0$ -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	46
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	46	1	1	1	1	-2

The intersection matrix for the  $P_1$ -curves (curves 7...14)

No	7	8	9	10	11	12	13	14
7	-2	1	1	1	0	1	0	20
8	1	-3	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	1
10	1	0	0	-2	1	0	0	0
11	0	0	0	1	-2	0	0	1
12	1	0	0	0	0	-2	1	0
13	0	0	0	0	0	1	-2	1
14	20	1	1	0	1	0	1	-2

The intersection matrix for the  $P_\infty$ -curves (curves 15...20)

No	15	16	17	18	19	20
15	-1	1	1	1	1	2
16	1	-8	0	0	0	1
17	1	0	-2	0	0	1
18	1	0	0	-4	0	1
19	1	0	0	0	-8	1
20	2	1	1	1	1	-1

The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{8,1}$  for  $n \leq 30$

Their basic properties:

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
21	1	1	1	0	0	0	-1
22	1	3	1	0	0	0	-1
23	9	1	12	0	0	0	0
24	9	3	12	0	0	0	0
25	17	1	18	1	1	0	0
26	17	3	18	1	1	0	0
27	25	1	30	2	0	2	4
28	25	3	30	2	0	2	4

Their intersection numbers with other curves:

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6
21	1	1	1	0	0	1	0	0	0
22	1	3	1	0	1	0	0	0	0
23	9	1	12	6	0	0	0	0	6
24	9	3	12	6	0	0	0	0	6
25	17	1	18	8	0	0	0	2	8
26	17	3	18	8	0	0	2	0	8
27	25	1	30	14	0	0	2	0	14
28	25	3	30	14	0	0	0	2	14

**b) Those with the curves over  $P_1$ :**

No	$n$	$k$	deg	7	8	9	10	11	12	13	14
21	1	1	1	0	1	0	0	0	0	0	0
22	1	3	1	0	0	1	0	0	0	0	0
23	9	1	12	4	0	0	0	0	0	0	4
24	9	3	12	4	0	0	0	0	0	0	4
25	17	1	18	6	0	0	0	0	0	0	6
26	17	3	18	6	0	0	0	0	0	0	6
27	25	1	30	10	0	0	0	0	0	0	10
28	25	3	30	10	0	0	0	0	0	0	10

**c) Those with the curves over  $P_\infty$ :**

No	$n$	$k$	deg	15	16	17	18	19	20
21	1	1	1	0	1	0	0	0	0
22	1	3	1	0	0	0	0	1	0
23	9	1	12	1	2	0	0	2	1
24	9	3	12	1	2	0	0	2	1
25	17	1	18	2	2	0	0	0	2
26	17	3	18	2	0	0	0	2	2
27	25	1	30	3	2	0	0	4	3
28	25	3	30	3	4	0	0	2	3

**d) Those of the Hecke curves with each other:**

No	$n$	$k$	deg	21	22	23	24	25	26	27	28
21	1	1	1	-1	0	0	0	0	0	0	0
22	1	3	1	0	-1	0	0	0	0	0	0
23	9	1	12	0	0	0	0	2	2	2	2
24	9	3	12	0	0	0	0	2	2	2	2
25	17	1	18	0	0	2	2	0	4	6	2
26	17	3	18	0	0	2	2	4	0	2	6
27	25	1	30	0	0	2	2	6	2	4	8
28	25	3	30	0	0	2	2	2	6	8	4