

Invariants of the Surface $\tilde{Z}_{8,3}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	1	22	24	-18	26	-2

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	\mathbb{L}_∞	\mathbb{L}	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	192	5	4	2	4	1	2	4	0	4	8	18	0	0

The Singularities of the associated singular surface $Z_{8,3}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 6]	-	2	1	[1, 6], [0, 1]	[37, 96, 64]	[5, -2, 13]
2	[2, 5]	-	2	2	[2, 5], [-1, -2]	[29, -192, 320]	[5, -2, 13]
3	[2, 7]	-	2	1	[2, 7], [-1, -3]	[53, -368, 640]	[5, 2, 13]
4	[3, 6]	-	2	2	[3, 14], [1, 5]	[205, 1168, 1664]	[5, 2, 13]

The CM-Singularities above P_1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
5	[1, 1]	+	1	1	[1, 1], [-1, 0]	[3, -24, 64]	[3, 0, 16]
6	[2, 3]	+	1	2	[2, 3], [-1, -1]	[19, -120, 192]	[3, 0, 16]

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
7	[1, 3]	-	2	3	[1, 3], [0, 1]	[13, 56, 64]	[4, 4, 13]
8	[1, 4]	-	2	3	[1, 4], [0, 1]	[21, 72, 64]	[4, 4, 13]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
9	[1, 0]	1	1	[8, 3]	2	[3, 3]
10	[1, 2]	1	2	[2, 1]	1	[2]
11	[1, 4]	1	3	[4, 3]	3	[2, 2, 2]
12	[3, 0]	1	4	[8, 3]	2	[3, 3]

The Basic Curves on $\tilde{Z}_{8,3}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	2	2	0	-2
6	2	2	0	-2
7	1	1	0	-2
14	1	1	0	-2
15	0	0	0	-2
24	0	0	0	-2

The intersection matrix for the non-exceptional curves:

No	1	6	7	14	15	24
1	-2	46	0	32	0	12
6	46	-2	32	0	12	0
7	0	32	-2	20	0	8
14	32	0	20	-2	8	0
15	0	12	0	8	-2	2
24	12	0	8	0	2	-2

The intersection matrix for the P_0 -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	46
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	46	1	1	1	1	-2

The intersection matrix for the P_1 -curves (curves 7...14)

No	7	8	9	10	11	12	13	14
7	-2	1	1	1	0	1	0	20
8	1	-3	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	1
10	1	0	0	-2	1	0	0	0
11	0	0	0	1	-2	0	0	1
12	1	0	0	0	0	-2	1	0
13	0	0	0	0	0	1	-2	1
14	20	1	1	0	1	0	1	-2

The intersection matrix for the P_∞ -curves (curves 15...24)

No	15	16	17	18	19	20	21	22	23	24
15	-2	1	0	1	1	0	0	1	0	2
16	1	-3	1	0	0	0	0	0	0	0
17	0	1	-3	0	0	0	0	0	0	1
18	1	0	0	-2	0	0	0	0	0	1
19	1	0	0	0	-2	1	0	0	0	0
20	0	0	0	0	1	-2	1	0	0	0
21	0	0	0	0	0	1	-2	0	0	1
22	1	0	0	0	0	0	0	-3	1	0
23	0	0	0	0	0	0	0	1	-3	1
24	2	0	1	1	0	0	1	0	1	-2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{8,3}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
25	3	1	4	0	0	0	-1
26	3	3	4	0	0	0	-1
27	11	1	12	1	1	0	0
28	11	3	12	1	1	0	0
29	19	1	20	2	1	1	2
30	19	3	20	2	1	1	2
31	27	1	36	5	1	4	8
32	27	3	36	5	1	4	8

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6
25	3	1	4	2	0	0	0	0	2
26	3	3	4	2	0	0	0	0	2
27	11	1	12	6	0	0	0	0	6
28	11	3	12	6	0	0	0	0	6
29	19	1	20	10	0	0	0	0	10
30	19	3	20	10	0	0	0	0	10
31	27	1	36	18	0	0	0	0	18
32	27	3	36	18	0	0	0	0	18

b) Those with the curves over P_1 :

No	n	k	deg	7	8	9	10	11	12	13	14
25	3	1	4	1	0	1	0	0	0	0	1
26	3	3	4	1	1	0	0	0	0	0	1
27	11	1	12	4	0	0	0	0	0	0	4
28	11	3	12	4	0	0	0	0	0	0	4
29	19	1	20	6	2	0	0	0	0	0	6
30	19	3	20	6	0	2	0	0	0	0	6
31	27	1	36	12	0	0	0	0	0	0	12
32	27	3	36	12	0	0	0	0	0	0	12

c) Those with the curves over P_∞ :

No	n	k	deg	15	16	17	18	19	20	21	22	23	24
25	3	1	4	0	0	1	0	0	0	0	1	0	0
26	3	3	4	0	1	0	0	0	0	0	0	1	0
27	11	1	12	1	0	1	0	0	0	0	1	0	1
28	11	3	12	1	1	0	0	0	0	0	0	1	1
29	19	1	20	2	0	1	0	0	0	0	1	0	2
30	19	3	20	2	1	0	0	0	0	0	0	1	2
31	27	1	36	3	2	1	0	0	0	0	1	2	3
32	27	3	36	3	1	2	0	0	0	0	2	1	3

d) Those of the Hecke curves with each other:

No	n	k	deg	25	26	27	28	29	30	31	32
25	3	1	4	-1	0	0	0	0	0	0	0
26	3	3	4	0	-1	0	0	0	0	0	0
27	11	1	12	0	0	0	2	2	2	4	2
28	11	3	12	0	0	2	0	2	2	2	4
29	19	1	20	0	0	2	2	2	4	6	6
30	19	3	20	0	0	2	2	4	2	6	6
31	27	1	36	0	0	4	2	6	6	8	10
32	27	3	36	0	0	2	4	6	6	10	8