

# Invariants of the Surface $\tilde{Z}_{8,5}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	1	22	24	-18	26	-2

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	192	5	4	2	4	1	2	4	0	4	8	18	0	0

## The Singularities of the associated singular surface $Z_{8,5}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[1, 6]	+	2	1	[1, 6], [0, 1]	[37, 96, 64]	[5, -2, 13]
2	[2, 5]	+	2	2	[2, 5], [-1, -2]	[29, -192, 320]	[5, -2, 13]
3	[2, 7]	+	2	1	[2, 7], [-1, -3]	[53, -368, 640]	[5, 2, 13]
4	[3, 6]	+	2	2	[3, 14], [1, 5]	[205, 1168, 1664]	[5, 2, 13]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
5	[1, 3]	+	2	1	[1, 3], [0, 1]	[13, 56, 64]	[4, 4, 13]
6	[1, 4]	+	2	1	[1, 4], [0, 1]	[21, 72, 64]	[4, 4, 13]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
7	[1, 1]	-	2	2	[1, 1], [-1, 0]	[3, -24, 64]	[3, 0, 16]
8	[2, 3]	-	2	2	[2, 3], [-1, -1]	[19, -120, 192]	[3, 0, 16]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
9	[1, 0]	1	1	[8, 5]	3	[2, 3, 2]
10	[1, 2]	1	2	[2, 1]	1	[2]
11	[1, 4]	1	3	[4, 1]	1	[4]
12	[3, 0]	1	4	[8, 5]	3	[2, 3, 2]

## The Basic Curves on $\tilde{Z}_{8,5}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	2	2	0	-2
6	2	2	0	-2
7	1	1	0	-2
14	1	1	0	-2
15	0	0	0	-2
24	0	0	0	-2

### The intersection matrix for the non-exceptional curves:

No	1	6	7	14	15	24
1	-2	46	0	32	0	12
6	46	-2	32	0	12	0
7	0	32	-2	20	0	8
14	32	0	20	-2	8	0
15	0	12	0	8	-2	2
24	12	0	8	0	2	-2

The intersection matrix for the  $P_0$ -curves (curves 1...6)

No	1	2	3	4	5	6
1	-2	1	1	1	1	46
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	46	1	1	1	1	-2

The intersection matrix for the  $P_1$ -curves (curves 7...14)

No	7	8	9	10	11	12	13	14
7	-2	1	1	1	0	1	0	20
8	1	-3	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	1
10	1	0	0	-2	1	0	0	0
11	0	0	0	1	-2	0	0	1
12	1	0	0	0	0	-2	1	0
13	0	0	0	0	0	1	-2	1
14	20	1	1	0	1	0	1	-2

The intersection matrix for the  $P_\infty$ -curves (curves 15...24)

No	15	16	17	18	19	20	21	22	23	24
15	-2	1	0	0	1	1	1	0	0	2
16	1	-2	1	0	0	0	0	0	0	0
17	0	1	-3	1	0	0	0	0	0	0
18	0	0	1	-2	0	0	0	0	0	1
19	1	0	0	0	-2	0	0	0	0	1
20	1	0	0	0	0	-4	0	0	0	1
21	1	0	0	0	0	0	-2	1	0	0
22	0	0	0	0	0	0	1	-3	1	0
23	0	0	0	0	0	0	0	1	-2	1
24	2	0	0	1	1	1	0	0	1	-2

The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{8,5}$  for  $n \leq 30$

Their basic properties:

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
25	5	1	6	0	0	0	-2
26	5	3	6	0	0	0	-2
27	13	1	14	0	0	0	-2
28	13	3	14	0	0	0	-2
29	21	1	32	3	1	2	4
30	21	3	32	3	1	2	4
31	29	1	30	4	2	2	4
32	29	3	30	4	2	2	4

Their intersection numbers with other curves:

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6
25	5	1	6	2	0	1	0	1	2
26	5	3	6	2	1	0	1	0	2
27	13	1	14	6	1	0	1	0	6
28	13	3	14	6	0	1	0	1	6
29	21	1	32	16	0	0	0	0	16
30	21	3	32	16	0	0	0	0	16
31	29	1	30	14	1	0	1	0	14
32	29	3	30	14	0	1	0	1	14

b) Those with the curves over  $P_1$ :

No	$n$	$k$	deg	7	8	9	10	11	12	13	14
25	5	1	6	2	0	0	0	0	0	0	2
26	5	3	6	2	0	0	0	0	0	0	2
27	13	1	14	4	1	1	0	0	0	0	4
28	13	3	14	4	1	1	0	0	0	0	4
29	21	1	32	10	1	1	0	0	0	0	10
30	21	3	32	10	1	1	0	0	0	0	10
31	29	1	30	10	0	0	0	0	0	0	10
32	29	3	30	10	0	0	0	0	0	0	10

c) Those with the curves over  $P_\infty$ :

No	$n$	$k$	deg	15	16	17	18	19	20	21	22	23	24
25	5	1	6	0	0	0	1	0	0	1	0	0	0
26	5	3	6	0	1	0	0	0	0	0	0	1	0
27	13	1	14	1	0	0	1	0	0	1	0	0	1
28	13	3	14	1	1	0	0	0	0	0	0	1	1
29	21	1	32	2	1	1	1	0	0	1	1	1	2
30	21	3	32	2	1	1	1	0	0	1	1	1	2
31	29	1	30	3	0	0	1	0	0	1	0	0	3
32	29	3	30	3	1	0	0	0	0	0	0	1	3

d) Those of the Hecke curves with each other:

No	$n$	$k$	deg	25	26	27	28	29	30	31	32
25	5	1	6	-2	0	0	0	0	0	0	0
26	5	3	6	0	-2	0	0	0	0	0	0
27	13	1	14	0	0	-2	0	2	2	4	6
28	13	3	14	0	0	0	-2	2	2	6	4
29	21	1	32	0	0	2	2	4	4	8	8
30	21	3	32	0	0	2	2	4	4	8	8
31	29	1	30	0	0	4	6	8	8	4	6
32	29	3	30	0	0	6	4	8	8	6	4