

# Invariants of the Surface $\tilde{Z}_{8,7}$

## Basic Numerical Invariants:

<b>Geometric:</b>	$p_g$	$h^{1,1}$	$b_2$	$sgn$	$c_2$	$K^2$
	2	30	34	-24	36	0

<b>Other:</b>	$m$	$g$	$r_0$	$g_0$	$r_1$	$g_1$	$s_{11}$	$r_\infty$	$g_\infty$	$h$	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	192	5	4	2	4	1	2	4	0	4	18	28	-2	-2

## The Singularities of the associated singular surface $Z_{8,7}$

### The Singularities above $P_0$ :

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
1	[0, 5]	-	2	1	[8, 5], [3, 2]	[89, 544, 832]	[1, 0, 64]
2	[0, 7]	-	2	2	[8, 7], [1, 1]	[113, 240, 128]	[1, 0, 64]
3	[4, 5]	-	2	1	[4, 5], [-1, -1]	[41, -144, 128]	[4, 4, 17]
4	[4, 7]	-	2	2	[4, 7], [1, 2]	[65, 288, 320]	[4, 4, 17]

### The CM-Singularities above $P_1$ (those of type $(-3)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
5	[1, 2]	+	2	1	[1, 2], [0, 1]	[7, 40, 64]	[7, -2, 7]
6	[2, 1]	+	2	1	[2, 1], [-1, 0]	[7, -40, 64]	[7, 2, 7]

### The anti-CM-Singularities above $P_1$ (those of type $(-2, -2)$ )

No	Name	Sign	Deg	Orbit	Basis of $M_P$	Quadratic Form	Reduced Form
7	[0, 1]	-	2	2	[0, 1], [-1, 0]	[1, -8, 64]	[1, 0, 48]
8	[0, 3]	-	2	2	[8, 3], [-3, -1]	[97, -568, 832]	[1, 0, 48]

### The Singularities above $P_\infty$ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
9	[1, 0]	1	1	[8, 7]	7	[2, 2, 2, 2, 2, 2]
10	[1, 2]	1	2	[2, 1]	1	[2]
11	[1, 4]	1	3	[4, 3]	3	[2, 2, 2]
12	[3, 0]	1	4	[8, 7]	7	[2, 2, 2, 2, 2, 2, 2]

## The Basic Curves on $\tilde{Z}_{8,7}$ :

### Table of the non-exceptional basic curves

No	$p_a$	$g$	$\delta_C$	$C^2$
1	2	2	0	-2
6	2	2	0	-2
7	1	1	0	-2
14	1	1	0	-2
15	0	0	0	-3
34	0	0	0	-3

### The intersection matrix for the non-exceptional curves:

No	1	6	7	14	15	34
1	-2	46	0	32	0	12
6	46	-2	32	0	12	0
7	0	32	-2	20	0	8
14	32	0	20	-2	8	0
15	0	12	0	8	-3	2
34	12	0	8	0	2	-3

**The intersection matrix for the  $P_0$ -curves (curves 1...6)**

No	1	2	3	4	5	6
1	-2	1	1	1	1	46
2	1	-2	0	0	0	1
3	1	0	-2	0	0	1
4	1	0	0	-2	0	1
5	1	0	0	0	-2	1
6	46	1	1	1	1	-2

**The intersection matrix for the  $P_1$ -curves (curves 7...14)**

No	7	8	9	10	11	12	13	14
7	-2	1	1	1	0	1	0	20
8	1	-3	0	0	0	0	0	1
9	1	0	-3	0	0	0	0	1
10	1	0	0	-2	1	0	0	0
11	0	0	0	1	-2	0	0	1
12	1	0	0	0	0	-2	1	0
13	0	0	0	0	0	1	-2	1
14	20	1	1	0	1	0	1	-2

**The intersection matrix for the  $P_\infty$ -curves (curves 15...34)**

No	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	
15	-3	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	2
16	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1
23	1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	1
24	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	1
27	1	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
34	2	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	1	-2	-3

**The Hecke curves  $T = T_{n,k}$  on  $\tilde{Z}_{8,7}$  for  $n \leq 30$**

Their basic properties:

No	$n$	$k$	deg	$p_a$	$g_T$	$\delta$	$T^2$
35	7	1	8	0	0	0	-2
36	7	3	8	0	0	0	-2
37	15	1	24	1	1	0	-2
38	15	3	24	1	1	0	-2
39	23	1	24	4	2	2	4
40	23	3	24	4	2	2	4

**Their intersection numbers with other curves:**

a) Those with the curves over  $P_0$ :

No	$n$	$k$	deg	1	2	3	4	5	6
35	7	1	8	4	0	0	0	0	4
36	7	3	8	4	0	0	0	0	4
37	15	1	24	12	0	0	0	0	12
38	15	3	24	12	0	0	0	0	12
39	23	1	24	12	0	0	0	0	12
40	23	3	24	12	0	0	0	0	12

b) Those with the curves over  $P_1$ :

No	$n$	$k$	deg	7	8	9	10	11	12	13	14
35	7	1	8	2	1	1	0	0	0	0	2
36	7	3	8	2	1	1	0	0	0	0	2
37	15	1	24	8	0	0	0	0	0	0	8
38	15	3	24	8	0	0	0	0	0	0	8
39	23	1	24	8	0	0	0	0	0	0	8
40	23	3	24	8	0	0	0	0	0	0	8

c) Those with the curves over  $P_\infty$ :

No	$n$	$k$	deg	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
35	7	1	8	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
36	7	3	8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
37	15	1	24	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	1
38	15	3	24	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	1
39	23	1	24	2	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
40	23	3	24	2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	2

d) Those of the Hecke curves with each other:

No	$n$	$k$	deg	35	36	37	38	39	40
35	7	1	8	-2	0	0	0	0	2
36	7	3	8	0	-2	0	0	2	0
37	15	1	24	0	0	-2	2	4	4
38	15	3	24	0	0	2	-2	4	4
39	23	1	24	0	2	4	4	4	6
40	23	3	24	2	0	4	4	6	4