Invariants of the Surface $\tilde{Z}_{9,1}$

Basic Numerical Invariants:

Geometric:

p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
1	26	28	-22	30	-6

Other:

m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
324	10	6	4	3	3	3	5	0	6	11	20	0	1

The Singularities of the associated singular surface $Z_{9,1}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	$[0,8]$	$+$	1	1	$[9,8],[1,1]$	$[145,306,162]$	$[1,0,81]$
2	$[1,6]$	$+$	2	2	$[1,6],[0,1]$	$[37,108,81]$	$[9,-6,10]$
3	$[3,8]$	$+$	2	2	$[3,8],[1,3]$	$[73,486,810]$	$[9,6,10]$
4	$[1,5]$	$-$	3	3	$[1,5],[0,1]$	$[26,90,81]$	$[5,4,17]$
5	$[2,7]$	$-$	3	3	$[2,7],[-1,-3]$	$[53,-414,810]$	$[2,2,41]$
6	$[4,8]$	$-$	3	3	$[4,17],[-1,-4]$	$[305,-1296,1377]$	$[5,-4,17]$

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
7	$[0,1]$	$+$	1	1	$[0,1],[-1,0]$	$[1,-9,81]$	$[1,1,61]$
8	$[2,3]$	$+$	2	2	$[2,3],[-1,-1]$	$[19,-135,243]$	$[7,-3,9]$
9	$[3,2]$	$+$	2	2	$[3,2],[1,1]$	$[19,135,243]$	$[7,3,9]$

The anti-CM-Singularities above P1 (those of type $(-2,-2)$)

– there are none of this type

The Singularities above P_∞:

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
10	$[1,0]$	1	1	$[9,1]$	1	$[9]$
11	$[1,3]$	2	2	$[3,1]$	1	$[3]$
12	$[1,6]$	2	2	$[3,1]$	1	$[3]$
13	$[2,0]$	1	3	$[9,7]$	4	$[2,2,2,3]$
14	$[4,0]$	1	4	$[9,4]$	4	$[3,2,2,2]$

The Basic Curves on $\tilde{Z}_{9,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	4	4	0	-3
8	4	4	0	-3
9	3	3	0	-1
13	3	3	0	-1
14	0	0	0	-2
26	0	0	0	-2

The intersection matrix for the non-exceptional curves:

No	1	8	9	13	14	26
1	−3	78	0	54	0	18
8	78	−3	54	0	18	0
9	0	54	−1	35	0	12
13	54	0	35	−1	12	0
14	0	18	0	12	−2	3
26	18	0	12	0	3	−2

The intersection matrix for the P_0-curves (curves 1...8)

No	1	2	3	4	5	6	7	8
1	−3	1	1	1	1	1	1	78
2	1	−2	0	0	0	0	0	1
3	1	0	−2	0	0	0	0	1
4	1	0	0	−2	0	0	0	1
5	1	0	0	0	−2	0	0	1
6	1	0	0	0	0	−2	0	1
7	1	0	0	0	0	0	−2	1
8	78	1	1	1	1	1	1	−3

The intersection matrix for the P_1-curves (curves 9...13)

No	9	10	11	12	13
9	−1	1	1	1	35
10	1	−3	0	0	1
11	1	0	−3	0	1
12	1	0	0	−3	1
13	35	1	1	1	−1

The intersection matrix for the P_∞-curves (curves 14...26)

No	14	15	16	17	18	19	20	21	22	23	24	25	26
14	−2	1	1	1	1	0	0	0	1	0	0	0	3
15	1	−9	0	0	0	0	0	0	0	0	0	0	1
16	1	0	−3	0	0	0	0	0	0	0	0	0	1
17	1	0	0	−3	0	0	0	0	0	0	0	0	1
18	1	0	0	0	−2	1	0	0	0	0	0	0	0
19	0	0	0	0	1	−2	1	0	0	0	0	0	0
20	0	0	0	0	0	1	−2	1	0	0	0	0	0
21	0	0	0	0	0	0	1	−3	0	0	0	0	1
22	1	0	0	0	0	0	0	0	−3	1	0	0	0
23	0	0	0	0	0	0	0	0	1	−2	1	0	0
24	0	0	0	0	0	0	0	0	0	1	−2	1	0
25	0	0	0	0	0	0	0	0	0	0	1	−2	1
26	3	1	1	1	0	0	0	1	0	0	0	1	−2

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{9,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
27	1	1	1	0	0	0	-1
28	4	4	6	0	0	0	-1
29	7	2	8	0	0	0	-2
30	10	1	18	0	0	0	-2
31	13	4	14	0	0	0	-2
32	16	2	24	0	0	0	-2
33	19	1	20	1	1	0	0
34	22	4	36	3	2	1	4
35	25	2	30	1	0	1	0
36	28	1	48	5	2	3	6

Their intersection numbers with other curves:
a) Those with the curves over P_0:

No	n	k	deg	1	2	3	4	5	6	7	8
27	1	1	1	0	1	0	0	0	0	0	0
28	4	4	6	3	0	0	0	0	0	0	3
29	7	2	8	4	0	0	0	0	0	0	4
30	10	1	18	8	0	1	1	0	0	0	8
31	13	4	14	6	0	1	1	0	0	0	6
32	16	2	24	12	0	0	0	0	0	0	12
33	19	1	20	10	0	0	0	0	0	0	10
34	22	4	36	18	0	0	0	0	0	0	18
35	25	2	30	14	0	1	1	0	0	0	14
36	28	1	48	24	0	0	0	0	0	0	24

b) Those with the curves over P_1:

No	n	k	deg	9	10	11	12	13
27	1	1	1	0	1	0	0	0
28	4	4	6	2	0	0	0	2
29	7	2	8	2	0	1	1	2
30	10	1	18	6	0	0	0	6
31	13	4	14	4	0	1	1	4
32	16	2	24	8	0	0	0	8
33	19	1	20	6	0	1	1	6
34	22	4	36	12	0	0	0	12
35	25	2	30	10	0	0	0	10
36	28	1	48	16	0	0	0	16

c) Those with the curves over P_∞:

No	n	k	deg	14	15	16	17	18	19	20	21	22	23	24	25	26
27	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
28	4	4	6	0	1	0	0	0	0	0	1	1	0	0	0	0
29	7	2	8	0	0	0	0	1	0	0	0	0	0	0	1	0
30	10	1	18	1	2	0	0	0	1	0	0	0	0	1	0	1
31	13	4	14	1	0	0	0	0	0	0	1	1	0	0	0	1
32	16	2	24	1	2	0	0	1	0	0	1	1	0	0	1	1
33	19	1	20	2	2	0	0	0	0	0	0	0	0	0	0	2
34	22	4	36	3	4	0	0	0	0	0	1	1	0	0	0	3
35	25	2	30	2	4	0	0	1	0	0	0	0	0	0	1	2
36	28	1	48	3	2	0	0	1	0	1	1	1	1	0	1	3

d) Those of the Hecke curves with each other:

No	n	k	deg	27	28	29	30	31	32	33	34	35	36
27	1	1	1	-1	0	0	0	0	0	0	0	0	0
28	4	4	6	0	-1	0	0	0	0	0	0	0	0
29	7	2	8	0	0	-2	0	0	0	0	2	0	0
30	10	1	18	0	0	0	-2	0	0	2	2	0	2
31	13	4	14	0	0	0	0	-2	0	2	2	2	2
32	16	2	24	0	0	0	0	0	-2	4	2	2	2
33	19	1	20	0	0	0	2	2	4	0	4	2	6
34	22	4	36	0	0	2	2	2	2	4	4	4	8
35	25	2	30	0	0	0	0	2	2	2	4	0	6
36	28	1	48	0	0	0	2	2	2	6	8	6	6