

Invariants of the Surface $\tilde{Z}_{9,2}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	2	32	36	-26	38	-2

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	L_∞	L	$2S_\infty$	$2S$
	324	10	6	4	3	3	0	5	0	6	16	28	-1	-1

The Singularities of the associated singular surface $Z_{9,2}$

The Singularities above P_0 :

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[1, 8]	+	1	1	[1, 8], [0, 1]	[65, 144, 81]	[2, 2, 41]
2	[2, 5]	+	2	2	[2, 5], [-1, -2]	[29, -216, 405]	[5, 4, 17]
3	[4, 7]	+	2	2	[4, 7], [1, 2]	[65, 324, 405]	[5, -4, 17]
4	[0, 5]	-	3	3	[9, 5], [-2, -1]	[106, -414, 405]	[1, 0, 81]
5	[3, 5]	-	3	3	[3, 5], [1, 2]	[34, 234, 405]	[9, 6, 10]
6	[4, 6]	-	3	3	[4, 15], [1, 4]	[241, 1152, 1377]	[9, -6, 10]

The CM-Singularities above P_1 (those of type (-3))

- there are none of this type

The anti-CM-Singularities above P_1 (those of type $(-2, -2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
7	[0, 4]	-	3	1	[9, 4], [2, 1]	[133, 549, 567]	[1, 1, 61]
8	[1, 2]	-	3	1	[1, 2], [0, 1]	[7, 45, 81]	[7, 3, 9]
9	[2, 1]	-	3	1	[2, 1], [-1, 0]	[7, -45, 81]	[7, -3, 9]

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
10	[1, 0]	1	1	[9, 2]	2	[5, 2]
11	[1, 3]	2	2	[3, 2]	2	[2, 2]
12	[1, 6]	2	2	[3, 2]	2	[2, 2]
13	[2, 0]	1	3	[9, 5]	2	[2, 5]
14	[4, 0]	1	4	[9, 8]	8	[2, 2, 2, 2, 2, 2, 2, 2]

The Basic Curves on $\tilde{Z}_{9,2}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	4	4	0	-3
8	4	4	0	-3
9	3	3	0	-2
16	3	3	0	-2
17	0	0	0	-3
34	0	0	0	-3

The intersection matrix for the non-exceptional curves:

No	1	8	9	16	17	34
1	-3	78	0	54	0	18
8	78	-3	54	0	18	0
9	0	54	-2	35	0	12
16	54	0	35	-2	12	0
17	0	18	0	12	-3	3
34	18	0	12	0	3	-3

The intersection matrix for the P_0 -curves (curves 1...8)

No	1	2	3	4	5	6	7	8
1	-3	1	1	1	1	1	1	78
2	1	-2	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	1
4	1	0	0	-2	0	0	0	1
5	1	0	0	0	-2	0	0	1
6	1	0	0	0	0	-2	0	1
7	1	0	0	0	0	0	-2	1
8	78	1	1	1	1	1	1	-3

The intersection matrix for the P_1 -curves (curves 9...16)

No	9	10	11	12	13	14	15	16
9	-2	1	0	1	0	1	0	35
10	1	-2	1	0	0	0	0	0
11	0	1	-2	0	0	0	0	1
12	1	0	0	-2	1	0	0	0
13	0	0	0	1	-2	0	0	1
14	1	0	0	0	0	-2	1	0
15	0	0	0	0	0	1	-2	1
16	35	0	1	0	1	0	1	-2

The intersection matrix for the P_∞ -curves (curves 17...34)

No	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
17	-3	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	3
18	1	-5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
20	1	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1
22	1	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	1
24	1	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	-5	0	0	0	0	0	0	0	0	1
26	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
34	3	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	1	-3

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{9,2}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
35	2	4	3	0	0	0	-1
36	5	1	6	0	0	0	-2
37	8	2	12	0	0	0	-2
38	11	4	12	1	1	0	0
39	14	1	24	1	1	0	-2
40	17	2	18	1	1	0	-2
41	20	4	36	1	1	0	-2
42	23	1	24	3	2	1	2
43	26	2	42	5	2	3	6
44	29	4	30	4	2	2	4

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8
35	2	4	3	1	1	0	0	0	0	0	1
36	5	1	6	2	0	1	1	0	0	0	2
37	8	2	12	6	0	0	0	0	0	0	6
38	11	4	12	6	0	0	0	0	0	0	6
39	14	1	24	12	0	0	0	0	0	0	12
40	17	2	18	8	0	1	1	0	0	0	8
41	20	4	36	18	0	0	0	0	0	0	18
42	23	1	24	12	0	0	0	0	0	0	12
43	26	2	42	20	0	1	1	0	0	0	20
44	29	4	30	14	0	1	1	0	0	0	14

b) Those with the curves over P_1 :

No	n	k	deg	9	10	11	12	13	14	15	16
35	2	4	3	1	0	0	0	0	0	0	1
36	5	1	6	2	0	0	0	0	0	0	2
37	8	2	12	4	0	0	0	0	0	0	4
38	11	4	12	4	0	0	0	0	0	0	4
39	14	1	24	8	0	0	0	0	0	0	8
40	17	2	18	6	0	0	0	0	0	0	6
41	20	4	36	12	0	0	0	0	0	0	12
42	23	1	24	8	0	0	0	0	0	0	8
43	26	2	42	14	0	0	0	0	0	0	14
44	29	4	30	10	0	0	0	0	0	0	10

c) Those with the curves over P_∞ :

No	n	k	deg	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
35	2	4	3	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
36	5	1	6	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
37	8	2	12	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
38	11	4	12	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
39	14	1	24	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	1
40	17	2	18	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1
41	20	4	36	2	1	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	2
42	23	1	24	2	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2
43	26	2	42	3	2	0	0	0	0	0	0	2	1	0	0	0	0	0	0	1	3
44	29	4	30	3	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3

d) Those of the Hecke curves with each other:

No	n	k	deg	35	36	37	38	39	40	41	42	43	44
35	2	4	3	-1	0	0	0	0	0	0	0	0	0
36	5	1	6	0	-2	0	0	0	0	0	0	0	0
37	8	2	12	0	0	-2	0	0	0	0	2	0	2
38	11	4	12	0	0	0	0	2	2	2	2	2	2
39	14	1	24	0	0	0	2	-2	0	0	2	4	4
40	17	2	18	0	0	0	2	0	-2	2	2	2	2
41	20	4	36	0	0	0	2	0	2	-2	4	6	6
42	23	1	24	0	0	2	2	2	2	4	2	6	4
43	26	2	42	0	0	0	2	4	2	6	6	6	6
44	29	4	30	0	0	2	2	4	2	6	4	6	4