Multiple description coding by successive quantization

Codage à description multiple par quantification successive

Saeed Moradi, Tamás Linder, and Saeed Gazor*

Two structured multiple description (MD) vector quantization schemes with an iterative technique for designing the codebooks and partitions are proposed. The schemes are derived from the recent theoretical work by Chen et al. In the first scheme, the central decoder is formed by the weighted sum of the side codebooks, whereas the second scheme employs the optimum central decoder. The objective of the proposed iterative method is to minimize a Lagrangian cost function (defined as the weighted sum of the central and side distortions) to jointly design the side codebooks and find the associated partitions. The optimal parameters for minimizing the central distortion are also found. Simulations demonstrate that the proposed methods achieve performance close to that of the unstructured, full-search MD quantizer with considerably less complexity and with only a few iterations.

Deux approches structurées de quantification vectorielle à description multiple (DM), basées sur une démarche itérative de la conception technique du livre de code (*codebook*) et des partitions, sont proposées en référence aux récents travaux théoriques de Chen et al. Dans le premier cas, le décodeur central est constitué de la somme pondérée des *codebooks* latéraux, tandis que la seconde approche emploie le régime optimal du décodeur central. L'objectif de la méthode itérative proposée est de réduire au minimum la fonction lagrangienne de coût (définie comme la somme pondérée des distorsions centrales et des côtés) pour concevoir conjointement les *codebooks* latéraux et les partitions associées. Les paramètres optimaux pour réduire au minimum la distorsion centrale sont également trouvés. Les simulations montrent que les méthodes proposées réalisent une performance proche de celle d'un quantificateur DM non structuré à pleine recherche avec beaucoup moins de complexité et peu d'itérations.

Keywords: codebook optimization; Lagrangian approach; multiple description vector quantization

I Introduction

Multiple description (MD) quantization is a promising approach for gaining robustness for transmission over a diversity communication system with several channels. In this technique, the source is quantized and mapped to a set of descriptions which are sent separately over multiple independent channels. Each description can be individually decoded with small degradation, but if all descriptions are available they can be jointly decoded to obtain a higher quality reconstruction. In the general two-description quantization problem, source samples are quantized into two descriptions with rates R_1 and R_2, respectively. The reconstructions of the received descriptions induce the side distortions D_1 and D_2, respectively; if both descriptions are received, the central distortion is D_0. In many applications, a balanced design is considered, where $R_1 = R_2$ and $D_1 = D_2$ [1]–[2].

The first constructive method towards multiple description scalar quantization (MDSQ) was proposed in [3] and [4]. The key component of this method is the index assignment, which maps an index to an index pair to be transmitted over two separate channels. The design of the index assignment is a difficult problem. The authors of [3] and [4] provided several heuristic methods for constructing balanced index assignments which are not necessarily optimal, but are likely to perform well. The analysis of this class of balanced quantizers reveals that asymptotically (at high rates) it is $3.07\,\mathrm{dB}$ away from the rate-distortion bound [5] in terms of the central and side distortion product when a uniform central quantizer is used. This granular distortion gap can be reduced by $0.4\,\mathrm{dB}$ when the central quantizer cells are better optimized [6].

The framework of MDSQ was later extended to multiple description lattice vector quantization (MDLVQ) in [7]. The design relies heavily on the choice of the lattice structure to facilitate the construction of index assignments. The analysis of these quantizers shows that the constructions are high-resolution optimal for asymptotically high dimensions. However, for lower dimensions, optimization of the code cells can also improve the high-resolution performance [8]–[9]. The major difficulty in constructing both MDSQ and MDLVQ is to find good index assignments, and thus the overall design would be simplified significantly if the index-assignment component could be eliminated altogether. Recently, Chen et al. [10] developed an MD quantization method via Gram-Schmidt orthogonalization that avoids the index-assignment problem. This technique promises to obtain the El Gamal-Cover (EGC) [2] achievable rate-distortion region for a memoryless Gaussian source at all rates, and for a memoryless non-Gaussian source at high rates, by subtractive dithering and successive quantization along with quantization splitting. Inspired by this interesting successive quantization scheme, we propose two practical multiple description quantization schemes with an iterative method to jointly design the codebooks so as to minimize a Lagrangian cost function that includes central and side distortions. We also find optimality conditions in order to obtain a joint codebook design algorithm for both proposed MD quantization schemes.

The rest of the paper is organized as follows. Section II explains the basic structure of the proposed MD quantizers. The design technique is discussed in Section III. Optimal transformations and parameters are

*Saeed Moradi is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario M5S 3G4, Canada. E-mail: saeed@comm.utoronto.ca. He performed the work reported in this paper while at Queen's University, Kingston, Ontario, Canada. Tamás Linder is with the Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada. E-mail: linder@mast.queensu.ca. Saeed Gazor is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, Ontario K7L 3N6, Canada. E-mail: s.gazor@queensu.ca. This paper was chosen as the Best Student Paper in the Communications and Networking Symposium at the 2008 IEEE Canadian Conference on Electrical and Computer Engineering. It is presented here in an expanded format.

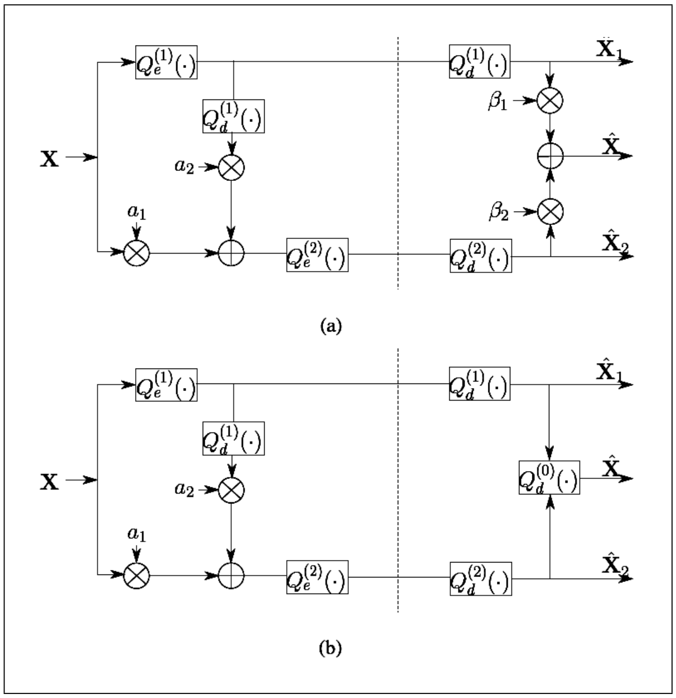

Figure 1: *MDVQ with two channels: (a) MDVQ-WSC scheme in which the central codebook is generated as the weighted sum of the two side codebooks; (b) MDVQ-OC scheme using the optimum central decoder.*

found in Section IV. Section V provides the complexity comparison. Simulation results are presented in Section VI. Finally, Section VII concludes the paper.

II Proposed MD quantization schemes

In [10] a structured MD entropy-constrained vector quantization (MD-ECVQ) scheme was proposed and shown to be asymptotically optimal within the limit of large block lengths. Our goal is to develop a method to iteratively optimize the structured quantization scheme proposed in [10] for practical quantizer dimensions, an issue that was not studied in [10]. The structure of the proposed MDVQ scheme is depicted in Fig. 1. The structures shown are similar to that of the asymptotically optimal MD-ECVQ scheme of [10], except that the dithered lattice quantizers are replaced by ordinary nearest-neighbour vector quantizers. These multiple description systems produce two different lossy descriptions of the source with quantizers $Q^{(1)}$ and $Q^{(2)}$. The input is a k-dimensional vector $\mathbf{X}$. The quantizer $Q^{(1)}$ uses a codebook $\mathbf{Y}=\{\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_N\}$, and the quantizer $Q^{(2)}$ uses a codebook $\mathbf{Z}=\{\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_M\}$. Both $Q^{(1)}$ and $Q^{(2)}$ are nearest-neighbour quantizers. Since we are interested in the balanced case where channels operate at equal rates, we take M to be equal to N, i.e., $M=N$. The encoders $Q_e^{(1)}$ and $Q_e^{(2)}$ generate indices i and j, which correspond to code vectors $\mathbf{y}_i$ and $\mathbf{z}_j$ respectively. In other words, if the input vector $\mathbf{x}$ lies in the central partition region W_{ij}, then indices i and j are generated. This input will be mapped to code vectors $\mathbf{y}_i$ and $\mathbf{z}_j$ at the first and second side decoder respectively. As a result, we can introduce new partitions of the input, each associated with a particular side quantizer as follows:

$$R_i=\bigcup_{m=1}^{M}W_{im},\quad S_j=\bigcup_{n=1}^{N}W_{nj}, \tag{1}$$

where R_i is the set of all input vectors mapped to the first side quantizer index i, and S_j is similarly the set of all input vectors mapped to the second side quantizer index j. The input to the second quantizer, $Q^{(2)}$, is produced by linear transformation (scaling) of the input vector $\mathbf{X}$ and $Q^{(1)}(\mathbf{X})$ (or equivalently $Q_d^{(1)}(Q_e^{(1)}(\mathbf{X}))$) with scalars a_1 and a_2. The indices i and j are transmitted over the separate channels provided by the diversity system. If only one of the indices is received, the corresponding side decoder is used to reconstruct the source vector. However, if both indices are received, the central decoder of the MDVQ with weighted sum central (MDVQ-WSC) decoder scheme reconstructs the source using linear transformations of the received decoded descriptions with transformation matrices β_1 and β_2. The optimized transformations a_1, a_2, β_1, and β_2 are discussed in Section IV. After the encoder and decoder of the quantizers are designed, the linear transformation can be replaced by the optimal MD decoder $Q^{(0)}$. If both indices are received by the MDVQ with optimum central (MDVQ-OC) decoder system, the decoder uses the optimum central codebook to reconstruct the source. This will improve the central decoder's performance at the cost of increased complexity.

III Design method

In this section, we present an iterative algorithm for designing the quantizers. The algorithm iteratively minimizes a Lagrangian cost function which includes constraints on the side distortions. This procedure leads to a possibly sub-optimal design of quantizers under the given constraints. The Lagrangian cost function is given by

$$\begin{aligned}L&=\lambda_0D_0+\lambda_1D_1+\lambda_2D_2\\&=\lambda_0E\left[\|\,\mathbf{X}-\beta_1Q^{(1)}(\mathbf{X})-\beta_2Q^{(2)}(\mathbf{X})\,\|^2\right]\\&\quad+\lambda_1E\left[\|\,\mathbf{X}-Q^{(1)}(\mathbf{X})\,\|^2\right]+\lambda_2E\left[\|\,\mathbf{X}-Q^{(2)}(\mathbf{X})\,\|^2\right],\end{aligned} \tag{2}$$

where λ_0, λ_1, and λ_2 are positive constants. The optimality conditions for minimizing the Lagrangian function are derived in the next section.

III.A Optimality conditions for the MDVQ-WSC system

For a fixed second side quantizer $Q^{(2)}$ and for a given first side quantizer $Q^{(1)}$ partition of the input space, the $Q^{(1)}$ codebook is optimal if, for each i, $\mathbf{y}_i$ minimizes the conditional Lagrangian function given the region R_i. As a result, the optimal $\mathbf{y}_i$ is the $\mathbf{y}$ that minimizes the conditional Lagrangian function

$$\begin{aligned}L_{1,i}&=\lambda_0E\left[\|\,\mathbf{X}-\beta_1\mathbf{y}-\beta_2Q^{(2)}(a_1\mathbf{X}+a_2\mathbf{y})\,\|^2|\,\mathbf{X}\in R_i\right]\\&\quad+\lambda_1E\left[\|\,\mathbf{X}-\mathbf{y}\,\|^2|\,\mathbf{X}\in R_i\right].\end{aligned} \tag{3}$$

Since $\mathbf{y}$ is an argument of the quantization function $Q^{(2)}$, an explicit minimization solution of the Lagrangian function turns out to be intractable. However, if we ease the notion of optimality as in [11] and fix $Q^{(2)}$, then the above Lagrangian function becomes quadratic in $\mathbf{y}$ and can be minimized with an iterative technique which takes the encoder of the second quantizer to be fixed while optimizing the decoder of the first quantizer. Thus, we seek $\mathbf{y}$ to minimize the Lagrangian function

$$\begin{aligned}L_{1,i}&=\lambda_0E\left[\|\,\mathbf{X}-\beta_1\mathbf{y}-\beta_2\mathbf{U}\,\|^2|\,\mathbf{X}\in R_i\right]\\&\quad+\lambda_1E\left[\|\,\mathbf{X}-\mathbf{y}\,\|^2|\,\mathbf{X}\in R_i\right],\end{aligned} \tag{4}$$

where $\mathbf{U}=Q^{(2)}(a_1\mathbf{X}+a_2\mathbf{y}_i)$. Taking the gradient of (4) with respect to $\mathbf{y}$ yields

$$\begin{aligned}\frac{\partial L_{1,i}}{\partial\mathbf{y}}&=-2\lambda_0\beta_1^TE\left[(\mathbf{X}-\beta_1\mathbf{y}-\beta_2\mathbf{U})\mid\mathbf{X}\in R_i\right]\\&\quad-2\lambda_1E\left[(\mathbf{X}-\mathbf{y})\mid\mathbf{X}\in R_i\right].\end{aligned} \tag{5}$$

The optimal $\mathbf{y}$ can then be found by solving the equation

$$\begin{aligned}\left(\lambda_0\beta_1^T\beta_1+\lambda_1I\right)\mathbf{y}&=\lambda_0\beta_1^TE\left[(\mathbf{X}-\beta_2\mathbf{U})\mid\mathbf{X}\in R_i\right]\\&\quad+\lambda_1E\left[\mathbf{X}\mid\mathbf{X}\in R_i\right],\end{aligned} \tag{6}$$

where I is the identity matrix. The solution of the above equation is given by

$$\mathbf{y}_i^* = \left(\lambda_0 \beta_1^T \beta_1 + \lambda_1 I\right)^{-1} \left(\lambda_0 \beta_1^T E\left[\left(\mathbf{X} - \beta_2 Q^{(2)}\left(a_1 \mathbf{X} + a_2 \mathbf{y}_i\right)\right) \mid \mathbf{X} \in R_i\right] + \lambda_1 E\left[\mathbf{X} \mid \mathbf{X} \in R_i\right]\right). \quad (7)$$

For a fixed $Q^{(1)}$ and for a given $Q^{(2)}$ partition of the input space, the $Q^{(2)}$ codebook is optimal if, for each j, $\mathbf{z}_j$ minimizes the conditional Lagrangian function in region S_j. Then, the optimal $\mathbf{z}_j$ is the $\mathbf{z}$ that minimizes the conditional Lagrangian function

$$L_{2,j} = \lambda_0 E\left[\| \mathbf{X} - \beta_1 Q^{(1)}(\mathbf{X}) - \beta_2 \mathbf{z} \|^2 \mid \mathbf{X} \in S_j\right] + \lambda_2 E\left[\| \mathbf{X} - \mathbf{z} \|^2 \mid \mathbf{X} \in S_j\right]. \quad (8)$$

Similarly, we seek $\mathbf{z}$ to minimize the Lagrangian function

$$L_{2,j} = \lambda_0 E\left[\| \mathbf{U}(\mathbf{X}) - \beta_2 \mathbf{z} \|^2 \mid \mathbf{X} \in S_j\right] + \lambda_2 E\left[\| \mathbf{X} - \mathbf{z} \|^2 \mid \mathbf{X} \in S_j\right], \quad (9)$$

where $\mathbf{U}(\mathbf{X}) = \mathbf{X} - \beta_1 Q^{(1)}(\mathbf{X})$. Taking the gradient of (9) with respect to $\mathbf{z}$ yields

$$\frac{\partial L_{2,j}}{\partial \mathbf{z}} = -2\lambda_0 \beta_2^T E\left[(\mathbf{U}(\mathbf{X}) - \beta_2 \mathbf{z}) \mid \mathbf{X} \in S_j\right] - 2\lambda_2 E\left[(\mathbf{X} - \mathbf{z}) \mid \mathbf{X} \in \mathbf{X}_j\right]. \quad (10)$$

The optimal $\mathbf{z}$ can then be found by solving the equation

$$\left(\lambda_0 \beta_2^T \beta_2 + \lambda_2 I\right) \mathbf{z} = \lambda_0 \beta_2^T E\left[\mathbf{U}(\mathbf{X}) \mid \mathbf{X} \in S_j\right] + \lambda_2 E\left[\mathbf{X} \mid \mathbf{X} \in S_j\right]. \quad (11)$$

The solution of the above equation is given by

$$\mathbf{z}_j^* = \left(\lambda_0 \beta_2^T \beta_2 + \lambda_2 I\right)^{-1} \times \left(\lambda_0 \beta_2^T E[\mathbf{X} - \beta_1 Q^{(1)}(\mathbf{X}) \mid \mathbf{X} \in S_j] + \lambda_2 E[\mathbf{X} \mid \mathbf{X} \in S_j]\right). \quad (12)$$

Equations (7) and (12) provide the design conditions required to improve the codebooks of the side quantizers in an iterative procedure in order to minimize the Lagrangian cost function.

III.B Optimality conditions for the MDVQ-OC system

The derivation of optimality conditions for the MDVQ-OC system is almost identical to the argument in the previous section. Since this scheme uses the optimum central decoder, the first terms of $L_{1,i}$ in (4) vanish, and the optimal $\mathbf{y}^*$ and $\mathbf{z}^*$ are found to be

$$\mathbf{y}_i^* = E\left[\mathbf{X} \mid \mathbf{X} \in R_i\right], \quad (13)$$

$$\mathbf{z}_j^* = E\left[\mathbf{X} \mid \mathbf{X} \in S_j\right]. \quad (14)$$

III.C Design algorithm

This section introduces an iterative technique to enhance the codebooks and, consequently, minimize the Lagrangian cost function as the optimization criterion. The iterative algorithm is similar to the Generalized Lloyd Algorithm (GLA) [12]. However, unlike the GLA, it does not necessarily produce a non-increasing sequence of Lagrangian values. Suppose we have a training set $\mathbf{T}$ that includes L training vectors $\mathbf{x}_l$, $l = 1, 2, \ldots, L$. We use the superscript (n) to indicate variables in the n-th iteration step. Suppose we have initial codebooks $\mathbf{Y}^{(0)}$ and $\mathbf{Z}^{(0)}$, obtained by traditional single-description design, for the first and second quantizer respectively. Let L_n denote the Lagrangian value computed in the n-th iteration step. The iterative algorithm steps are as follows:

1. **Encode and partition training set:** Encode each vector in the training set with the current codebooks. Let $i(k)$ and $j(k)$ denote the indices generated in encoding vector $\mathbf{x}_k \in \mathbf{T}$. Compute the Lagrangian cost L_{n+1}.
2. **Termination test:** If $|L_n - L_{n+1}|/L_n < \delta$, where δ is a fixed small positive threshold, or if n exceeds the maximum number of desired steps, terminate the algorithm.
3. **Update the $Q^{(1)}$ codebook:** Replace each code vector in the first side quantizer codebook by the conditional centroid according to (7) and (13) for MDVQ-WSC and MDVQ-OC respectively in order to obtain the new codebook $\mathbf{Y}^{(n+1)}$.
4. **Encode and repartition training set:** Produce a new set of indices $i(k)$ and $j(k)$ according to the updated codebook $\mathbf{Y}^{(n+1)}$.
5. **Update the $Q^{(2)}$ codebook:** Replace each code vector in the second side quantizer codebook by the conditional centroids given in (12) and (14) for MDVQ-WSC and MDVQ-OC respectively to obtain the new codebook $\mathbf{Z}^{(n+1)}$. Go back to step 1.

Since the encoder generates optimal partitions only by jointly searching the codebooks, it may rarely happen that the Lagrangian value increases in an iteration. The possibility of a non-monotonic Lagrangian sequence raises the issue of how to effectively terminate the iterative process. Similarly to the remedy proposed in [11], the termination step may be modified so that the algorithm will terminate when the relative change in L_n is less than δ for several consecutive steps, or when the total number of algorithm steps has reached a given limit. Another consequence of the non-monotonicity in L_n is that the final-stage codebooks at termination may not be the best ones. This problem is easily resolved by choosing the codebooks from an intermediate iteration with the lowest L_n.

Once the encoder and decoder of the MDVQ-WSC quantizers are found by the proposed iteration technique, the central decoder can be replaced by the optimal MD decoder. In other words, given received code vectors $\mathbf{y}_i$ and $\mathbf{z}_j$ from the first and second channels respectively, the optimal central decoder $Q_d^{(0)}$ reconstructs the central description as $Q_d^{(0)}(\mathbf{y}_i, \mathbf{z}_j) = E[\mathbf{X} \mid \mathbf{X} \in W_{ij}]$. According to our simulation results, this adjustment yields better performance by the central decoder.

IV Optimal transformations and parameters

IV.A Optimal a_i

Assuming a balanced case, we must choose a_1 and a_2 carefully such that they lead to balanced side distortions, $D_1 \approx D_2$. We investigated the effect of a_1 and a_2 on the side distortions. Fig. 2 shows the side distortions as the coefficient a_2 increases and a_1 is kept fixed at $a_1 = -1$ for $k = 4$ and $R = 0.5$ bits per source sample (bpss). The source is a unit-variance memoryless Gaussian source. The side distortion of the first quantizer remains almost constant, while the second side distortion changes slightly with various values of a_1 and a_2.

IV.B Optimal β_i

We can easily derive the optimal transformations β_1 and β_2 by minimizing the central distortion, $D_0 = E[\| \mathbf{X} - \beta_1 \hat{\mathbf{X}}_1 - \beta_2 \hat{\mathbf{X}}_2 \|^2]$, with respect to the matrices β_1 and β_2. Using the orthogonality principle, the optimal β_1 and β_2 must satisfy

$$E\left[\left(\mathbf{X} - \beta_1 \hat{\mathbf{X}}_1 - \beta_2 \hat{\mathbf{X}}_2\right) \hat{\mathbf{X}}_1^T\right] = 0, \quad (15)$$

$$E\left[\left(\mathbf{X} - \beta_1 \hat{\mathbf{X}}_1 - \beta_2 \hat{\mathbf{X}}_2\right) \hat{\mathbf{X}}_2^T\right] = 0. \quad (16)$$

If we define β to equal $[\beta_1 \; \beta_2]$ and $\hat{\mathbf{X}}^T$ to equal $[\hat{\mathbf{X}}_1^T \; \hat{\mathbf{X}}_2^T]$, then we can rewrite (15) and (16) as $E[(\mathbf{X} - \beta \hat{\mathbf{X}}) \hat{\mathbf{X}}^T] = 0$. As a result, β can be found by solving $\beta E[\hat{\mathbf{X}} \hat{\mathbf{X}}^T] = E[\mathbf{X} \hat{\mathbf{X}}^T]$, which yields

$$\beta = E\left[\mathbf{X} \hat{\mathbf{X}}^T\right] \left(E\left[\hat{\mathbf{X}} \hat{\mathbf{X}}^T\right]\right)^{-1}.$$

IV.C Choosing Lagrangian multipliers λ_i

We introduced an iteration technique in the previous section in order to minimize the Lagrangian cost function for a fixed set of Lagrangian

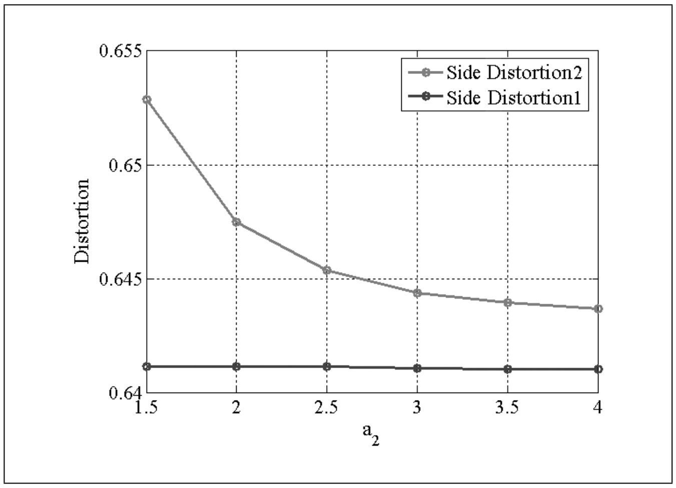

Figure 2: *Distortion of the side decoders as the coefficient a_2 increases ($a_1 = -1$, $k = 4$, and $R = 0.5$ bpss).*

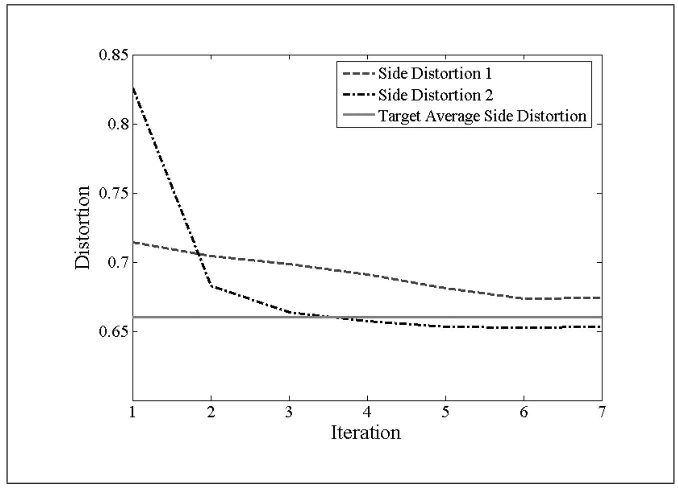

Figure 3: *Effect of tuning the Lagrangian multipliers on side distortions for a memoryless Gaussian input source with unit variance and target side distortions $D_{1,t} = D_{2,t} = 0.66$ ($k = 4$ and $R = 0.5$ bpss).*

multipliers λ_i, $i = 0, 1, 2$. For the target side distortions, another problem remains; namely, to find the optimal λ_i^*, $i = 0, 1, 2$, that leads the side distortions to converge to the desired target side distortion. In fact, each set of $(\lambda_0, \lambda_1, \lambda_2)$ corresponds to a single point on the convex hull of an MD achievable distortion region. This implies that as the values of λ_i, $i = 0, 1, 2$, change, we have a tradeoff between central and side distortions. Therefore, appropriate selection of λ_i leads to the desired target distortions at the side decoders. The search for $(\lambda_0, \lambda_1, \lambda_2)$ is somewhat analogous to the search for the appropriate value of λ, or equivalently the slope of the rate-distortion function, in the design of an entropy-constrained vector quantizer [13]. For ECVQ, [13] proposes a bisection approach to facilitate the code design for a particular desired rate. The ECVQ algorithm designs a vector quantizer for a specific λ at the middle of a range $[\lambda_{\min}, \lambda_{\max}]$. The design process then shortens this range to the lower or higher half in the direction that decreases the gap between the obtained and desired rates. Now consider the Lagrangian function introduced in (2). For balanced distortions where $\lambda_1 = \lambda_2 = \tilde{\lambda}$ and $D_s = 1/2(D_1 + D_2)$, the Lagrangian function in (2) can be rewritten as

$$\begin{aligned} L &= \lambda_0 D_0 + \tilde{\lambda}(D_1 + D_2) \\ &= \lambda_0 D_0 + 2\tilde{\lambda} D_s. \end{aligned} \tag{17}$$

Since only the relative values of the Lagrangian multipliers are meaningful [14], we can divide (17) by λ_0. Then the Lagrangian function reduces to

$$L = D_0 + \lambda D_s, \tag{18}$$

where $\lambda = 2\tilde{\lambda}/\lambda_0$. As shown in [15], a small value of λ leads to a higher D_s, and a large value of λ leads to a smaller D_s. Thus, we can modify the iterative technique of the previous section as follows. Similarly to the approach proposed in [13], we limit the value of λ to the range $[0, 1]$ and set $\lambda = 0.5$ as the initial value. We then observe the obtained average side distortion D_s at the end of each iteration. If the obtained D_s is higher than the target side distortion, we simply shorten the range of λ to the higher half. Similarly, if the obtained D_s is lower than the target side distortion, we shorten the range of λ to the lower half. It should be noted that obtaining a D_s lower than the target side distortion is not necessarily desired since it leads to a higher central distortion. For instance, if the observed D_s is higher than the target side distortion at the end of the first iteration, we update λ to 0.75, which is the middle of the range $[0.5, 1]$. Alternatively, consider the obtained Lagrangian function at the end of the n-th iteration as

$$L_n = \lambda_{0,n} D_{0,n} + \lambda_{1,n} D_{1,n} + \lambda_{2,n} D_{2,n}. \tag{19}$$

For the target side distortions $D_{1,t}$ and $D_{2,t}$, we propose to modify the Lagrangian multipliers of (19) as follows:

$$\tilde{\lambda}_{i,n+1} = \lambda_{i,n} \frac{D_{i,n}}{D_{i,t}}, \quad i = 1, 2, \tag{20}$$

$$\tilde{\lambda}_{0,n+1} = \lambda_{0,n}. \tag{21}$$

Assuming that the sum of multipliers is one, we normalize $\tilde{\lambda}_{i,n+1}$, $i = 0, 1, 2$, as

$$\lambda_{i,n+1} = \frac{\tilde{\lambda}_{i,n+1}}{\sum_{j=0}^{2} \tilde{\lambda}_{j,n+1}}, \quad i = 0, 1, 2. \tag{22}$$

In this way L_n remains a convex combination of individual distortions. In general, (20) simply scales $\lambda_{i,n}$, $i = 1, 2$, proportionally to the ratio of the observed corresponding side distortion to the target side distortion, and (22) normalizes the resulting $\tilde{\lambda}_{i,n+1}$, $i = 0, 1, 2$. As a result, this may lead to the faster convergence of λ_i, $i = 0, 1, 2$, to the optimal values. This simple procedure allows us to efficiently control the tradeoff between the central and side distortions. Fig. 3 demonstrates the effect of tuning the Lagrangian multipliers according to (20)–(22) on the observed side distortions for a four-dimensional memoryless unit-variance Gaussian input source and target side distortions $D_{1,t} = D_{2,t} = 0.66$ with rate $R = 0.5$ bpss.

Table 1

Comparison of computational complexity and memory requirements for k-dimensional input vector ($R_1 = R_2 = R$ bpss, $N = 2^{kR}$)

	Computational complexity	Memory requirement
Optimum MDVQ	$N^2 k$	$N^2 k + 2N^2$
MDVQ-OC	$N^2 + 4Nk$	$4Nk + N^2$
MDVQ-WSC	$4Nk$	$4Nk + 2N$

V Complexity and memory requirements

In this section the computational complexity and memory requirements of our proposed methods are compared with those of Vaishampayan's potentially optimum MD vector quantizer [3]. The scheme proposed in [3] is a very general form of MDVQ that can, if its parameters are appropriately chosen, achieve optimal performance for a given quantizer dimension. However, because of its general structure, the scheme is rather complex. Also, in practice its optimality is not guaranteed, as the iterative algorithm for its design ensures only convergence to a locally optimum solution. The computational complexity, which is the number of multiplication and addition operations,

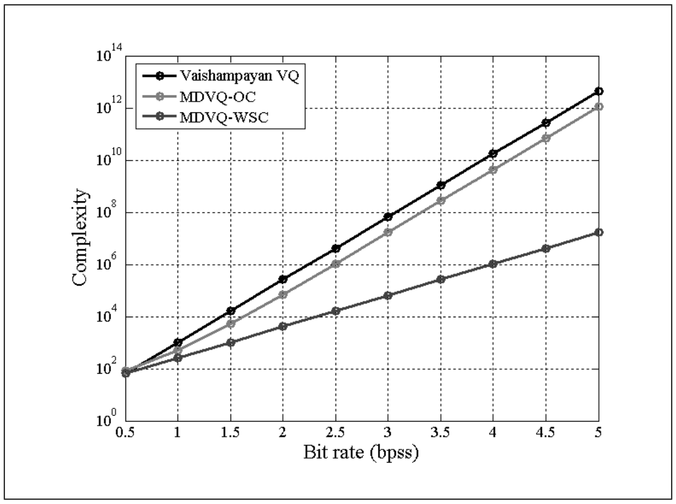

Figure 4: *The complexity comparison as a function of bit rate for four-dimensional source vector.*

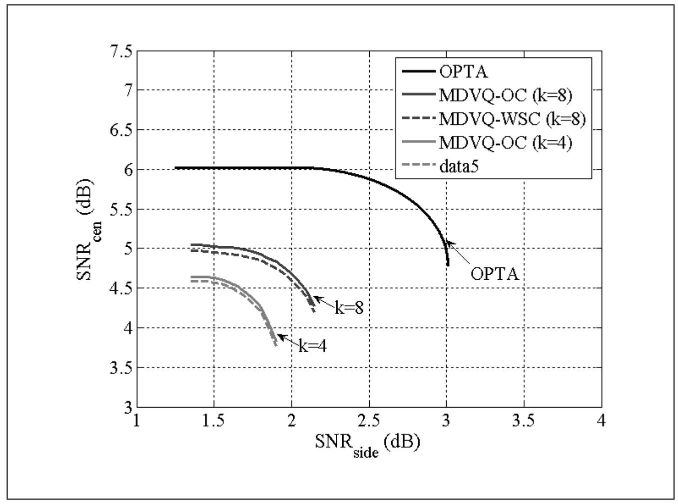

Figure 5: *MDVQ for unit-variance memoryless Gaussian 4- and 8-dimensional source vectors at $R = 0.5$ bpss for various values of λ.*

Table 2

Selected performance results for quantizer dimension k with correlation coefficient ρ and rates $R_1 = R_2 = 0.5$ bpss

ρ	k	$\mathrm{SNR}_{\mathrm{side}}$ (dB)	$\mathrm{SNR}_{\mathrm{cen}}$ (dB) MDVQ-WSC	MDVQ-OC	Optimum MDVQ
0.0	4	1.65	4.47	4.52	4.55
0.0	8	1.75	4.89	4.96	5.01
0.9	4	6.15	8.13	8.21	8.29
0.9	8	7.45	9.82	9.91	10.01

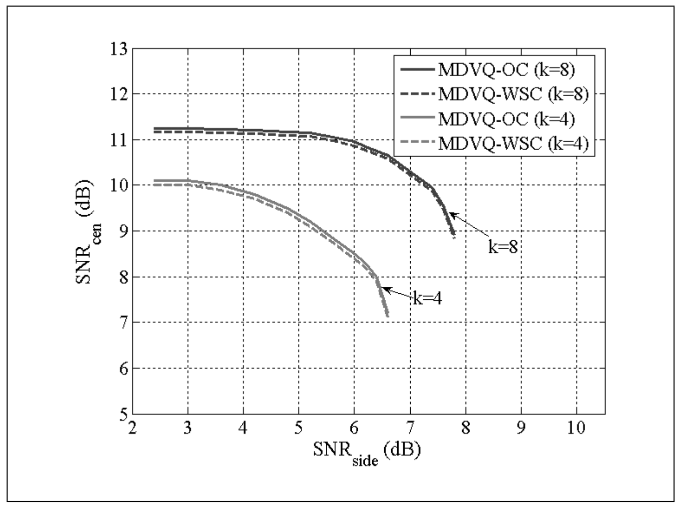

Figure 6: *MDVQ for unit-variance Gauss-Markov 4- and 8-dimensional source vectors with $\rho = 0.9$ at $R = 0.5$ bpss for various values of λ.*

and the memory requirements are compared in Table 1. The number of operations required to encode each k-dimensional input vector in a conventional vector quantizer depends mainly on the calculation of distortion between two vectors and is given by N^2k operations. In our proposed methods, (7) and (12) for MDVQ-WSC and (13) and (14) for MDVQ-OC give the number of required operations, which are $4Nk$ and $N^2 + 4Nk$ respectively. Fig. 4 also shows the complexity comparison as the bit rate increases. It is clear that our proposed methods, and in particular the MDVQ-WSC method, can significantly reduce the computational complexity and memory requirements when a squared-error distortion measure is used for distortion.

VI Simulation results

Simulation results are provided in this section for the MDVQ-WSC and MDVQ-OC schemes with two channels for a zero-mean unit-variance stationary first-order Gauss-Markov source with correlation coefficient ρ. The encoding rates are set to $R_1 = R_2 = 0.5$ bpss. Block sizes $k = 4$ and $k = 8$ are considered. We have also set $\lambda_1 = \lambda_2 = \lambda$ in all the results presented here. A training set of length 50 000 source vectors was used along with a termination threshold of 0.001 in all cases.

Initialization of the design algorithm is an important issue when one seeks to obtain an initial set of codebooks. The first applied initialization technique selects the codebook obtained by uniform partitioning of the training set. We have also used two other initialization techniques reported in [3]. Neither technique achieves results that are uniformly better than the other's. The presented simulation results are the best that have been obtained using all three initialization techniques.

Figs. 5 and 6 show the simulation results for $\rho = 0$ (memoryless Gaussian source) and $\rho = 0.9$ (highly correlated Gauss-Markov source). We have plotted the SNR for the central decoder, $\mathrm{SNR}_{\mathrm{cen}} = 10\log_{10}(1/D_0)$, as a function of $\mathrm{SNR}_{\mathrm{side}} = 10\log_{10}(2/(D_1 + D_2))$, for various values of λ. The optimum rate-distortion bound is the same as that given in [3].

Table 2 shows selected performance results for a memoryless Gaussian source as well as for a Gauss-Markov source with $\rho = 0.9$ and $R_1 = R_2 = 0.5$ bpss. These results are compared with the best experimental results achieved by Vaishampayan's optimum MDVQ in [3]. As can be seen from the table, the performance of our proposed schemes is very close to that of the optimum MDVQ. Simulation results also reveal the significant improvement in performance obtained by increasing the block size k in all cases, as expected from the known property of vector quantization. An increase in block size results in a greater improvement for $\rho = 0.9$ than for $\rho = 0$. For instance, for a memoryless Gaussian source, only 0.15 dB are gained when the block size is increased from $k = 4$ to $k = 8$ at $\mathrm{SNR}_{\mathrm{side}} = 1.5$ dB for the MDVQ-WSC scheme. However, for a Gauss-Markov source with correlation coefficient $\rho = 0.9$, a gain of 1.15 dB is achieved for the same increase in the block size from $k = 4$ to $k = 8$ at $\mathrm{SNR}_{\mathrm{side}} = 3.0$ dB. This result indicates the significance of increasing the block size for highly correlated sources such as speech and video. Since the tradeoff between central and side distortion can be carefully controlled by selecting appropriate values for λ_1, λ_2, and λ_0, our approach provides the designer with greater design flexibility.

Table 3

Gain achieved by replacing linear-transformation central decoder with optimum central decoder in MDVQ-WSC ($R_1 = R_2 = 0.5$ bpss)

	Gauss-Markov		Uniform	Laplacian
	$\rho = 0.0$	$\rho = 0.9$		
$k = 4$	0.11 dB	0.14 dB	2.3 dB	0.34 dB
$k = 8$	0.15 dB	0.17 dB	2.8 dB	0.41 dB

As we mentioned in Section III.C, after the encoder and decoder of the quantizers of the MDVQ-WSC scheme are designed, the linear-transformation central decoder can be replaced by the optimal MD decoder. Doing so results in better performance for the central decoder of the MDVQ-WSC scheme. Table 3 summarizes the gain achieved by replacing the central decoder of MDVQ-WSC with the optimum central decoder for various source distributions. Since, according to heuristic considerations, the linear transformation performs nearly as well as the optimum decoder for Gaussian sources, the achieved gain for Gaussian sources is expected to be negligible. However, since the output of side decoders is not Gaussian, a small gain is observed. On the other hand, for uniform source distribution, this gain is significant.

VII Conclusion

We proposed two successive multiple description quantization schemes with an iterative method to jointly design the codebooks by minimizing a Lagrangian cost function. This Lagrangian function includes central and side distortions. We also found optimality conditions and obtained a joint codebook design algorithm for the proposed MDVQ schemes. The proposed MD vector quantization schemes have relatively low complexity and, for moderately large dimensions, still perform comparably to the more complex, potentially optimal, unstructured MD vector quantization scheme in [3].

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Saeed Moradi received his B.Sc. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 2003, and the M.Sc. degree in electrical and computer engineering from Queen's University in Kingston, Ontario, Canada, in 2007. He is currently pursuing the Ph.D. degree in the Department of Electrical and Computer Engineering of the University of Toronto, Toronto, Ontario, Canada. From 2003 to 2005, he was involved in the design and development of high-tech software and hardware industrial projects in the telecommunications field. He joined the MERIT team in the IPSI Department, Fraunhofer Institute, Darmstadt, Germany, in summer 2004, and did research in video watermarking. His current research interests are in the area of wireless communication systems with an emphasis on cooperative communications and multiple-antenna (MIMO) systems.

Tamás Linder (S'92-M'93-SM'00) was born in Budapest, Hungary, in 1964. He received the M.S. degree in electrical engineering from the Technical University of Budapest in 1988, and the Ph.D. degree in electrical engineering from the Hungarian Academy of Sciences in 1992. He was a post-doctoral researcher at the University of Hawaii in 1992 and a Visiting Fulbright Scholar at the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, during 1993–1994. From 1994 to 1998 he was a faculty member in the Department of Computer Science and Information Theory at the Technical University of Budapest. From 1996 to 1998 he was also a visiting research scholar in the Department of Electrical and Computer Engineering, University of California, San Diego. In 1998 he joined Queen's University in Kingston, Ontario, Canada, where he is now a professor of mathematics and engineering in the Department of Mathematics and Statistics. His research interests include communications and information theory, source coding and vector quantization, machine learning, and statistical pattern recognition. Dr. Linder received the Premier's Research Excellence Award of the Province of Ontario in 2002 and the Chancellor's Research Award of Queen's University in 2003. He was an associate editor for source coding of the *IEEE Transactions on Information Theory* in 2003–2004.

Saeed Gazor (S'94-M'95-SM'98) received the B.Sc. degree in electronics engineering in 1987 and the M.Sc. degree in communication systems engineering in 1989, from Isfahan University of Technology, both *summa cum laude*, the highest honours. He received his Ph.D. in signal and image processing from Telecom Paris, Département Signal (École Nationale Supérieure des Télécommunications/ENST, Paris), France, in 1994. Since 1999, he has been on the faculty at Queen's University in Kingston, Ontario, Canada, where he currently holds the position of professor in the Department of Electrical and Computer Engineering. He is also cross-appointed to the Department of Mathematics and Statistics at Queen's University. Prior to joining Queen's University, he was an assistant professor in the Department of Electrical and Computer Engineering, Isfahan University of Technology, from 1995 to 1998. He was a research associate at the University of Toronto, Toronto, Ontario, Canada, from January 1999 to July 1999. His main research interests are array signal processing, statistical and adaptive signal processing, speech processing, image processing, detection and estimation theory, MIMO communication systems, collaborative networks, channel modelling, and information theory. His research has earned him a number of awards, including a provincial Premier's Research Excellence Award (now known as the Early Researcher Award), a Canadian Foundation for Innovation Award, and an Ontario Innovation Trust Award. Dr. Gazor is a member of Professional Engineers Ontario and is currently serving as an associate editor for the *IEEE Signal Processing Letters*.