ERRATA IN A QUANTIFICATION OF LONG TRANSIENT DYNAMICS

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Abstract. Our paper in SIAM J. Appl. Math. 82(2), 2022, pp. 381-407, has an incorrect statement in Theorem 6.3 and 6.4 in page 395. There is also an error in Example 6.13 in page 402. This document provides the corrected statements (corrections are in red).

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In this document, unchanged parts from the original manuscript are in blue, corrections/additions are in red. The correct statement of Theorem 6.3 should be:

THEOREM 6.3. *Let $v \in C^1(\mathbb{R}^n, \mathbb{R})$, $\alpha \in \mathbb{R}$ and $\alpha \neq 0$. If ξ is a v-transient center of (4.1), then the following hold:*

1. *$\phi(t, 0, \xi)$ is also a v-transient center for all $t \in \mathbb{R}^+$.*
2. *ξ is also an αv-transient center.*
3. *If $Dv(\phi(t, 0, \xi)) = 0$ for all $t < 0$, $\phi(t, 0, \xi)$ is also a v-transient center for all $t \in \mathbb{R}$.*

Proof for 3. With the assumption, we know that $T_s(\phi(T; 0, \xi)) > 0$ for given $T > 0$. The remaining proof is similar to 1. ∎

This corrects and clarifies that $t < 0$ an additional assumption is required that $Dv(\phi(t, 0, \xi)) = 0$ for all $t < 0$. The reason being ξ being transient center implies that $Dv(\phi(t, 0, \xi)) = 0$ for all $t > 0$ but not for $t < 0$. It is thus a necessary assumption to make. Beside that, the prove will be identical to 1.

As a result of this change, Theorem 6.4 needs to also be changed:

THEOREM 6.4. *Let $v \in C^1(\mathbb{R}^n, \mathbb{R})$. If ξ is a v-transient center, then for all $n = 1, 2, \ldots$ and $t \in \mathbb{R}^+$, $D^n v(\phi(t; 0, \xi)) = 0$.*

The statement under the Remark 6.5 should change to:

The definition of the set Ξ in the original manuscript is correct, but we clarify the discussion by adding more details. The next part of the text appears in page 396 and correction is shown in red.

Theorem 6.4 provides a necessary condition for a point to be v-transient center. To find these transient centers, good candidates would be trajectories on set $\{\nabla v \cdot f = 0\}$. Namely it is the set

$$\{\xi : \nabla v \cdot f(\phi(t; 0, \xi)) = 0 \text{ for all } t \in \mathbb{R}^+\}.$$

However it is convenient to work with the following subset since it is often important to consider the reversed flow as well.

$$\Xi = \{\xi : \nabla v \cdot f(\phi(t; 0, \xi)) = 0 \text{ for all } t \in \mathbb{R}\}. \tag{6.1}$$

Finally, there is an error in Example 6.13. Here we provide the full Example with corrections in red.

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Example 6.4. Consider the following system with a simple saddle fixed point at the origin.

$$\begin{cases} x' = x \\ y' = -y. \end{cases} \tag{0.1}$$

Let $v = x$. We now apply Theorem 6.12 to show that $\xi = (0, 0)$ is a v-transient center.

Proof. It is easy to see that $\phi^t\xi = (0, 0)$. Let $R > 0$. To show condition (1), we note that $\nabla \cdot f = 0$. Thus we can take $\beta = 0$ and condition (1) is satisfied. To show condition (2), let $\alpha = 2 > \beta$. Since $Dv = x$ we get,

$$2Dv(\nabla Dv \cdot f) - \alpha Dv^2 = 2x^2 - \alpha x^2 = 0.$$

Thus condition (2) is also satisfied.

To show condition (3), we note that

$$L = \left\| \mathrm{Jac}(f) \right\|_{op} = \left\| \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\|_{op} = 1.$$

Let $\eta = 2 = \frac{\alpha - \beta}{L}$. For all $r \in (0, R)$, observe that $\mathrm{vol}_{B_r(\xi)}(0) = \pi r^2$. Switching from Cartesian (x, y) to polar coordinates (s, θ) we get that $Dv = x = s\cos(\theta)$ and,

$$I_{B_r(\xi)}(0) = \int_0^r \int_0^{2\pi} Dv^2 s\, d\theta ds = \int_0^r \int_0^{2\pi} (s\cos(\theta))^2 s\, d\theta ds = \frac{\pi r^4}{4}.$$

Therefore we have that

$$\frac{I_{B_r(\xi)}(0)}{\mathrm{vol}_{B_r(\xi)}(0)} = \frac{1}{4}r^2 = \frac{1}{4}r^\eta$$

Thus condition (3) is satisfied. Thus from Theorem 6.12, we conclude that $\xi = (0, 0)$ is a $v-$transient center of this system. □