

Week #2 - Limits, Continuity, and the Derivative

Section 2.4

From “Calculus, Single Variable” by Hughes-Hallett, Gleason, McCallum et. al.

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SUGGESTED PROBLEMS

1. (a) As the cup of coffee cools, the temperature decreases, so $f'(t)$ is negative.
(b) Since $f'(t) = dH/dt$, the units are degrees Celsius per minute. The quantity $f'(20)$ represents the rate at which the coffee is cooling, in degrees per minute, 20 minutes after the cup is put on the counter.
4. (a) The statement $f(5) = 18$ means that when 5 milliliters of catalyst are present, the reaction will take 18 minutes. Thus the units for 5 are ml while the units for 18 are minutes.
(b) As in part (1), 5 is measures in ml. Since f' tells how fast T changes per unit a , we have f' measured in minutes/ml. If the amount of catalyst increases by 1 ml (from 5 to 6 ml), the reaction time decreases by about 3 minutes.
9. The units of $f'(x)$ are feet/mile. The derivative, $f'(x)$, represents the rate of change of elevation with distance from the source, so if the river is flowing downhill everywhere, the elevation is always decreasing and $f'(x)$ is always negative. (In fact, there may be some stretches where the elevation is more and less constant, so $f'(x) = 0$.)
11. (a) Since $W = f(c)$ where W is weight in pounds and c is the number of Calories consumed per day:
 - $f(1800) = 155$ means that consuming 1800 Calories per day results in a weight of 155 pounds
 - $f'(2000) = 0$ means that consuming close to 2000 Calories per day leads to same final body weight.
 - $f^{-1}(162) = 2200$ means that a weight of 162 is caused by a consumption of 2200 Calories per day(b) The units of dW/dc are pounds/(Calories/day).

QUIZ PREPARATION PROBLEMS

13. Since $f(t) = 1.15(1.014)^t$, we have

$$f(6) = 1.15(1.014)^6 = 1.25 \text{ billion people}$$

Interpretation: in $1993 + 6 = 1999$, the population of China will be approximately 1.25 billion people.

To estimate $f'(6)$, we use a small interval around 6:

$$\begin{aligned}
f'(6) &\simeq \frac{f(6.001) - f(6)}{f.006 - 6} \\
&= \frac{1.15(1.014)^{6.001} - 1.15(1.014)^6}{0.001} \\
&\simeq 0.0174 \text{ billion people per year}
\end{aligned}$$

In 1999, the population of China is growing at roughly 0.0174 billion people per year, or 17.4 million people per year.

16. (a) The pressure is in dynes/cm² at a depth of 100 meters.
(b) The depth of water in meters giving a pressure of 1.2×10^6 dynes/cm².
(c) The pressure at a depth of h meters plus a pressure of 20 dynes/cm².
(d) The pressure at a depth of 20 meters below the diver.
(e) The rate of increase of pressure with respect to depth, at 100 meters, in units of dynes/cm² per meter. Approximately, $p'(100)$ represents the increase in pressure in going from 100 meters to 101 meters.
(f) the depth, in meters, at which the rate of change of pressure with respect to depth is 20 dynes/cm² per meter.
17. Units of $g'(55)$ are mpg/mph. The statement $g'(55) = -0.54$ means that at 55 miles per hour the fuel efficiency (in miles per gallon, or mpg) of the car decreases at a rate of approximately one half mpg as the velocity increases by one mph.
21. (a) The units of compliance are units of volume per units of pressure, or liters per centimeter of water.
(b) The increase in volume for a 5 cm reduction in pressure is largest between 10 and 15 cm. Thus the compliance appears maximum between 10 and 15 cm of pressure reduction. The derivative is given by the slope, so

$$\text{Compliance} \simeq \frac{0.70 - 0.49}{15 - 10} = 0.042 \text{ liters per centimeter}$$

- (c) When the lung is nearly full, it cannot expand much more to accommodate more air.