SUGGESTED PROBLEMS

In Exercises 1-4, let $F(x) = \int_{0}^{x} f(t) \, dt$. Graph $F(x)$ as a function of $x$.

1. By the Fundamental Theorem, $f(x) = F'(x)$. Since $f$ is positive and increasing, $F$ is increasing and concave up. Since $F(0) = \int_{0}^{0} f(t)\,dt = 0$, the graph of $F$ must start from the origin.

2. Since $f$ is positive and decreasing and $F' = f$, $F$ is increasing and concave down. As with #1, the graph of $F$ must start from the origin.

3.
Since $f$ is always positive, $F$ is always increasing. $F$ has an inflection point where $f' = 0$. As with #1, the graph of $F$ must start from the origin.

4.

Since $f$ is always non-negative, $F$ is increasing. $F$ is concave up where $f$ is increasing and concave down where $f$ is decreasing; $F$ has inflection point at the critical points of $f$. As with #1, $F(0) = 0$.

**QUIZ PREPARATION PROBLEMS**

12. The graph of the derivative $F'$ of some function $F$ is given in Figure 6.27. If $F(20) = 150$, estimate the maximum value attained by $F$.

Since we are shown the derivative graph, the maximum value will occur when $F' = 0$, which occurs at $x = 50$. We can confirm that this is a local max by the first derivative test, and a global max because the $F'$ graph tells us that $F$ is increasing up to $x = 50$ and decreasing afterward.

To estimate the value of $F(50)$, we use the fundamental theorem, and the information that $F(20) = 150$:

$$F(50) - F(20) = \int_{20}^{50} F'(x) \, dx$$

Approximate the integral with a (very rough) estimate of the area under the graph:

$\approx 350$

So $F(50) \approx F(20) + 350 = 500$
We estimate the maximal value of $F$ on the interval shown is 500. Depending on your estimate of the area, your answer may differ significantly.

Find the derivatives in Problems 15-20.

15. $\frac{d}{dx} \int_0^x \cos(t^2) \, dt$
   Since the integral is in the form required by the Construction Theorem, we can simply state that
   
   $$\frac{d}{dx} \int_0^x \cos(t^2) \, dt = \cos(x^2)$$

19. $\frac{d}{dx} \int_x^1 \ln t \, dt$
   In this case, the $x$ variable is in the wrong end of the limit for the Construction Theorem to apply, so we have to swap the limits of integration first.
   
   $$\frac{d}{dx} \int_x^1 \ln t \, dt = \frac{d}{dx} \left[ - \int_1^x \ln t \, dt \right] = -\ln x$$

21. Let $g(x) = \int_0^x f(t) \, dt$. Using Figure 6.28, find
   
   (a) $g(0)$
   (b) $g'(1)$
   (c) The interval where $g$ is concave up.
   (d) The value of $x$ where $g$ takes its maximum on the interval $0 \leq x \leq 8$.

   ![Figure 6.28](image)

   (a) $g(0) = \int_0^0 f(t) \, dt$. Since the upper and lower limits of integration are the same, there is no area, so the integral is zero. Thus $g(0) = 0$.
   (b) The graph is the graph of $f = g'$, therefore $f(1) = -2$ implies that $g'(1) = -2$.
   (c) $g$ is concave up when $g' = f$ is increasing. This occurs for $1 \leq x \leq 6$.
   (d) $g$ is decreasing for $0 \leq x \leq 3$. $x = 3$ is a local minimum. For $3 \leq x \leq 8$, $g$ is increasing. Thus $x = 3$ is the global minimum of $g$ on the interval. This tells us that there are no local maxima within the interval to consider.

   To the find the global maximum then, we must look at the end points, and since the area under $g'$ to the right of $x = 3$ is larger than the area to the left, $g$ increases more to the right, so $g(8)$ will be the global maximum of $g$ on the given interval.