Last name:(blockletters)____________________ **First/Given Name:**________________

Student Number:____________________

MATH 121 - TEST 1 (Based on Assignments 1, 2, and 3)
Version 3B Fall 2010

This test consists of 3 questions to be answered in the space provided.
Show all work and give explanations when needed.

1. Animals introduced into new environments will experience exponential population growth, if they are well-adapted to their new surroundings. Observing such a new species in Ontario, biologists note that the population is growing by 25% per year, from an initial population of 120 individuals.

   (a) Write a formula for the population P of this new species.

   (b) What will the population be after 5 years? Include units in your answer.

   (c) How long will it take for the population to reach 10,000 individuals? Include units in your answer.

   (d) Find the doubling time for the population. Include units in your answer.

a) $P = 120(1+0.25)^t = 120(1.25)^t$

b) at $t=5$, $P = 120(1.25)^t \cong 366$ individuals

c) to reach 10,000 individuals,

$$10{,}000 = 120(1.25)^t$$

$$\ln\left[\frac{10{,}000}{120}\right] = \ln\left[(1.25)^t\right] \Rightarrow \ln\left(\frac{10\,000}{120}\right) = t\ln(1.25)$$

$$t = \frac{\ln\left(\frac{10000}{120}\right)}{\ln(1.25)} \cong 19.8 \text{ years}$$

d) The doubling time is t for which $P = 2P_0 = 2\cdot 120 = 240.$

$$240 = 120(1.25)^t$$

$$\ln\left[\frac{240}{120}\right] = \ln\left[1.25^t\right] \Rightarrow t = \frac{\ln(2)}{\ln(1.25)} \cong 3.1 \text{ years}$$

The exponential model $120e^{.25t}$ would also be a reasonable answer since we often use this model for population. On the midterm, I will say specifically whether growth is continuous (eg. pop., radioactivity) or not (eg. yearly interest)

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Test 1, Version 3B

2. (a) Evaluate the following limit.

$$\lim_{x\to 1} \frac{x^2+x-2}{x-1}$$

(b) Find a value of k for which the limit below exists. Once you have found the value, confirm your answer by evaluating the limit using your specific k value.

$$\lim_{x\to 2} \frac{x^2-2x+k}{x-2}$$

a) $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$ is form $\frac{0}{0}$: need to simplify to find true value

$= \lim_{x\to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x\to 1} (x+2) = 3$

$x \neq 1$

b) $\lim_{x\to 2} \frac{x^2-2x+k}{x-2}$ exists only if we can cancel $(x-2)$ in the denominator

$(x^2-2x+k) = (x-2)(x \quad) \Rightarrow k=0$

Confirm:

If $k=0$, $\lim_{x\to 2} \frac{x^2-2x}{(x-2)} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{x-2}}$

$= \lim_{x\to 2} x = 2 \Rightarrow$ limit exists if $k=0$.

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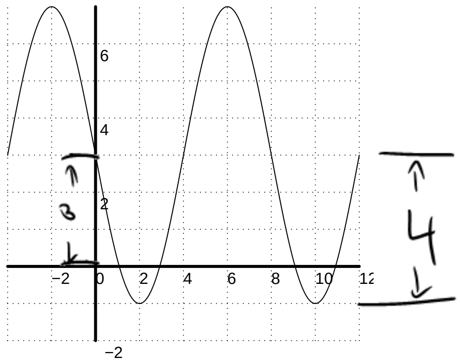
Test 1, Version 3B

(a) Find the derivative of $y = 10 + \cos(3x^2 - 1)$

(b) Give a formula for the oscillating function shown in the graph.
(Note: parts (a) and (b) are unrelated to each other.)



a) $y' = \frac{dy}{dx} = -\sin(3x^2 - 1) \cdot 6x$

$= -6x \sin(3x^2 - 1)$

b) amplitude is 4

center is shifted up by 3

period is 8

Form is sine graph, but flipped vertically

$y = 3 - 4\sin\left(\frac{2\pi}{8} x\right)$

$= 3 - 4\sin\left(\frac{\pi}{4} x\right)$