Last name:(blockletters)______________________ **First/Given Name:**_________________

Student Number:______________________

MATH 121 - TEST 2 (Based on Assignments 4, 5, 6 and 7)
Version 1A Fall 2010

This test consists of 3 questions to be answered in the space provided.
Show all work and give explanations when needed.

1. (a) Find the local linearization of $f(x) = e^{-x}$ around $x = 0$.
   (b) Use your answer from part (a) to find an approximation for $g(x) = e^{-2x}$ near $x = 0$.
   (c) Without referring to your previous answers, find the *quadratic* Taylor polynomial that approximates $g(x) = e^{-2x}$ near $x = 0$.

a) $f(x) = e^{-x} \rightarrow f(0) = 1$

$f'(x) = -e^{-x} \rightarrow f'(0) = -1$

So the linearization for $f(x)$ around $x=0$ is

$$y = 1 - x$$

b) since $g(x) = e^{-2x} = (e^{-x})^2 = [f(x)]^2$

$g(x) \cong (1-x)^2 = 1-2x+x^2$ for x near 0

Note: we also accepted

$e^{-x} \approx 1-x$

so

$e^{-2x} \approx 1-2x$

every x replaced w/ $2x$

c) $g(x) = e^{-2x} \rightarrow g(0) = 1$

$g'(x) = -2e^{-2x} \rightarrow g'(0) = -2$

$g''(x) = 4e^{-2x} \rightarrow g''(0) = 4$

So $$e^{-2x} \approx \underbrace{1}_{g(0)} \underbrace{-2}_{g'(0)}(x-0) + \underbrace{\frac{4}{2}}_{\frac{g''(0)}{2}}(x-0)^2$$

$$= 1-2x+2x^2$$

Note that this equals the approximation we found in b) through an alternate method.

2. Consider the family of functions

$$h(x) = 5(1 - e^{-kx})$$

where k is a positive constant, and $x \geq 0$.

(a) Find $h(0)$.

(b) Find $h'(0)$.

(c) Find $\lim_{x\to\infty} h(x)$.

(d) Find the intervals on which $h(x)$ is increasing, and those on which it is decreasing.

(e) Using the information from parts (a-d), sketch two members of the family, using $k = 1$ and $k = 2$. Identify which graph used which k value, and remember that $x \geq 0$.

a) $h(0) = 5(1-e^{0}) = 5(1-1) = 0$

b) $h'(x) = 5(+Ke^{kx})$

$= 5Ke^{kx}$

so $h'(0) = 5K$

c) $\lim_{x\to\infty} h(x) = \lim_{h\to\infty} 5(1-\underset{\to 0}{e^{-kx}}) = 5(1) = 5$

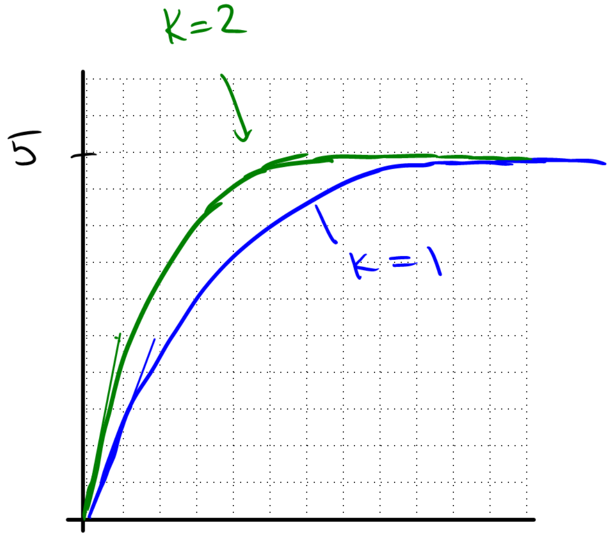
d) $h'(x) = 5Ke^{kx}$

Since $K>0$ (given in question) and $e^{kx} > 0$ (exp'l)

$h'(x)$ is always positive

$\Rightarrow$ $h(x)$ is increasing for all x.

It is never decreasing



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Test 2- 1A

3. You want to build a closed cylinder with a surface area of $A = 50\pi$ cm^2. The radius of the cylinder will be r cm, and the height h cm.

   Find the dimensions of such a cylinder for which the **volume is maximized**. Report the radius, height and the volume of the resulting cylinder.
   (You should show that the dimensions you find produce a **local** maximum for volume; you do *not* need argue why your answer produces a **global** maximum.)

$A =$ ends + sides $= 2(\pi r^2) + 2\pi r h$

$V = \pi r^2 h$

Solve for h in terms of r using A:

$$50\cancel{\pi} = 2\cancel{\pi} r^2 + 2\cancel{\pi} r h$$

$$50 - 2r^2 = 2rh$$

$$h = \frac{50 - 2r^2}{2r} = \frac{25 - r^2}{r}$$

so $V = \pi r^2 \left(\frac{25 - r^2}{r}\right) = 25\pi r - \pi r^3$

ind critical points of $V(r)$:

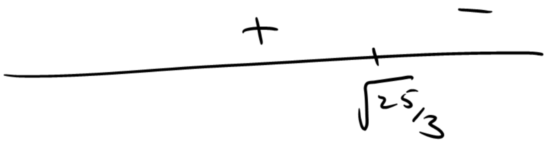
$$V' = 25\pi - 3\pi r^2$$

Set $=0$ to find critical points

$$0 = 25\cancel{\pi} - 3\cancel{\pi} r^2$$

$r^2 = \frac{25}{3}$, so $r = \pm\sqrt{\frac{25}{3}}$. we ignore neg. radius, as it is unacceptable in this problem

Sign of V'



By first derivative test, this value produces a local maximum for volume

$\Rightarrow$ $r = \sqrt{\frac{25}{3}} \approx 2.88$ cm $\quad h = \frac{25 - r^2}{r} = \frac{25 - 25/3}{\sqrt{25/3}} = \frac{50/3}{\sqrt{25/3}} \cong 5.77$ cm

and $V = \pi r^2 h \approx 150.35$ cm^3

is the largest can volume possible, given Area $= 50\pi$ cm^2