Last name:(blockletters)____________________ **First/Given Name:**________________

Student Number:____________________

MATH 121 - TEST 2 (Based on Assignments 4, 5, 6 and 7)
Version 3A Fall 2010

This test consists of 3 questions to be answered in the space provided.
Show all work and give explanations when needed.

1. A Boeing 747 aircraft traveling at speed s (in km/h) can travel $g(s)$ kms per liter of fuel. In particular, when traveling at $s = 900$ km/h, the plane can travel $g(900) = 0.08$ km per liter of fuel, and $g'(900) = -0.0004$.

   We now define $f(s)$ as the gas consumption (in l/km) of the aircraft.

   (a) Write $f(s)$ in terms of $g(s)$.

   (b) Find the values of $f(900)$ and $f'(900)$. Include units in your answers.

a) since $g(s)$ is consumption in km/l, and $f(s)$ is consumption in l/km,

$$f(s) = \frac{1}{g(s)}$$

b) $$f(900) = \frac{1}{g(900)} = \frac{1}{0.08} = 12.5 \ l/km$$

$$f'(s) = \frac{d}{ds}\left([g(s)]^{-1}\right) = -1[g(s)]^{-2} \cdot g'(s)$$

$$= \frac{-g'(s)}{[g(s)]^2}$$

so $$f'(900) = \frac{-g'(900)}{[g(900)]^2} = \frac{-(-0.0004)}{[0.08]^2} \approx 0.0625 \ \frac{l/km}{km/hr}$$

2. Consider the curve defined by the relation $x^2 + y^2 + ax + 3y = 7$ where a is a positive constant.

   (a) Find $\frac{dy}{dx}$.

   (b) Under what conditions on x and/or y will the tangent line to this curve be **horizontal**?

   (c) Under what conditions on x and/or y will the tangent line to this curve be **vertical**?

a) Take implicit $\frac{d}{dx}$ of both sides:

$$2x + 2y\frac{dy}{dx} + a + 3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y+3) = -2x-a$$

$$\frac{dy}{dx} = \frac{-2x-a}{2y+3}$$

b) Tangent lines will be horizontal when $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 0 = \frac{-2x-a}{\cancel{2y+3}} \quad \text{or} \quad \boxed{x = -a/2}$$

c) Tangent lines will be vertical when the denominator of $\frac{dy}{dx}$ is zero, or

$$2y + 3 = 0$$

$$\boxed{y = -3/2}$$

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3. The total revenue (in thousands of dollars) generated by selling q items is given by the formula

$$R(q) = \frac{q-5}{10+q}, \text{ given } q \geq 5$$

(a) Using the formula for the total revenue for q units given above, write a formula for $A(q)$, the **average revenue per unit** (thousands of dollars/unit)

(b) Find the value of q for which the average revenue per unit is maximized.

a) $A(q) = \dfrac{\text{total revenue}}{\#\text{ units}} = \dfrac{\frac{q-5}{10+q}}{q} = \dfrac{q-5}{(10+q)(q)}$ or $\dfrac{q-5}{10q+q^2}$

b) $A(q)$ will be highest at a critical point;

Setting $A'=0$,

$$A' = \frac{(1)(10q+q^2) - (q-5)(10+2q)}{(10q+q^2)^2} = 0$$

$$\cancel{10q} + q^2 - \cancel{10q} + 50 - 2q^2 + 10q = 0$$

$$-q^2 + 10q + 50 = 0 \quad \leftarrow \text{quadratic}$$

$$q = \frac{-10 \pm \sqrt{100 - 4(50)(-1)}}{-2} = 5 \pm \frac{\sqrt{300}}{2} \cong 13.6 \text{ or } -3.6$$

unit

not feasible value for q ↓

(c) Part of the graph of $R(q)$ is shown on the graph below. Indicate the maximum value you found from part (a), and clearly show how the average revenue for that value of q is represented on the graph.