Diophantine Equations

Diophantine Equations: These are polynomial equations for which the solutions are to be integers.

– They were named after Diophantos of Alexandria (Egypt), who wrote around 250 A.D. an extremely interesting treatise entitled The Arithmetica which consisted of 13 books. Of these, 6 survived in Greek throughout the ages. Recently (1976), an Arabic translation of 4 more books was discovered in a library in Persia.

– Perhaps the most notorious of all Diophantine equations is the equation

$$x^n + y^n = z^n. \tag{1}$$

For $n = 2$, infinitely many solutions exist; these are usually called pythagorean triplets because of the connection with the rule of Pythagoras (ca. 700 B.C.) for right-angled triangles. For example:

$$3^2 + 4^2 = 5^2.$$

However, already the Babylonians (ca. 1700 B.C.) knew how to generate pythagorean triplets systematically. (→ clay tablets)

Around 1637 P. Fermat asserted that the equation (1) has no (positive) integer solutions for $n \geq 3$. (Equivalently: no n-th power can be a sum of two n-th powers for $n \geq 3$.) He also asserted that he had a "marvellous proof" for this fact, but that the margin (of his copy of the Arithmetica) was too small to contain it.

Despite the attempts of many mathematicians (and lay people), this assertion remained unproven for many years and was referred to as Fermat's Last Theorem.

Recently in 1993/5, A. Wiles succeeded in proving Fermat's assertion (and much more)[1]. Indeed, it made newspapers headlines in June 1993, but then a gap in the argument was discovered, which he then subsequently repaired. The complete proof was published in 1995, and uses very advanced mathematics. (It also requires the earlier ideas and results of G.Frey (1986) and K. Ribet (1989)).

[1] In May 1997, A. Wiles received an honourary doctorate from Queens's for his work.