The GCD-Criterion and its Consequences

Theorem 4 (GCD-criterion): Let $m, n, c \in \mathbb{Z}$ be non-zero integers. Then the equation

$$mx + ny = c \tag{1}$$

has an integer solution (x, y) if and only if

$$\gcd(m, n) \mid c. \tag{2}$$

Corollary 1: $\gcd(m, n) = 1 \Leftrightarrow$ there exits $x, y \in \mathbb{Z}$ such that $mx + ny = 1$.

Corollary 2: If $g = \gcd(m, n)$, then $\gcd(\frac{m}{g}, \frac{n}{g}) = 1$.

Corollary 3: $\gcd(mk, nk) = \gcd(m, n) \cdot k$, if $k > 0$.

Corollary 4 (Euclid): If $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.

Corollary 4′ (Euclid): If $\gcd(a, b) = 1$, then $a \mid c$ and $b \mid c$ if and only if $ab \mid c$.

Corollary 5: If $\gcd(a, b) = 1$, then

$$\gcd(a, bc) = \gcd(a, c).$$