How to Solve $mx + ny = c$

Step 0: Use the Euclidean algorithm to find

$$g := \gcd(m, n).$$

Step 1: Use the GCD-criterion to check whether the equation

(1) $$mx + ny = c$$

has any solutions; i.e. test whether

(2) $$g := \gcd(m, n) \mid c.$$

If false, then equation (1) has no integer solutions.

Step 2: Use the method of back-substitution in the Euclidean algorithm to find integers (x_0, y_0) such that

(3) $$mx_0 + ny_0 = g.$$

Step 3: The general solution (x, y) of (1) is given by

(4) $$\left.\begin{aligned} x &= \frac{c}{g}x_0 + \frac{n}{g}t \\ y &= \frac{c}{g}y_0 - \frac{m}{g}t \end{aligned}\right\} \text{ where } t \in \mathbb{Z}.$$

Step 4: If applicable, analyze the constraints.
(E.g. $x \geq 0$, $y \geq 0 \rightsquigarrow$ inequalities for t.)