

# Some Unsolved Conjectures about Primes

Goldbach's Conjecture (Goldbach, 1742):

(G) Every integer  $n > 5$  is the sum of three primes.



(G') Every even integer  $n \geq 4$  is the sum of two primes.

Known: - every sufficiently large odd integer  $n$  is the sum of three primes. ( $n > 3^{3^{15}}$ )  
(Hardy/Littlewood, 1923; Vinogradov, 1937)  
- every integer is the sum of at most 19 primes.  
(Riesel/Vaughan, 1983)

Twin Primes Conjecture: There exist infinitely many twin primes, i.e. primes  $p$  such that  $p + 2$  is also prime.

Examples: (3, 5), (5, 7), (11, 13), (17, 19), ...,  
(1949, 1951), ...,  $107570463 \times 10^{2250} \pm 1$   
(Dubner, 1985), ...

Reference: P. Ribenboim, The Book of Prime Number Records, Springer-Verlag, New York, 1989.