The Fundamental Theorem of Arithmetic

Theorem 7 ("Building Block Theorem"): Every integer $n > 1$ is a product of prime numbers:

$$n = p_1 \cdot p_2 \cdot \ldots \cdot p_r, \tag{1}$$

where $p_1 \leq p_2 \leq \ldots \leq p_r$ are primes.

Theorem 8 ("Fundamental Theorem of Arithmetic"): Every integer $n > 1$ can be uniquely expressed as a product of prime numbers (up to order):

$$n = p_1 \cdot p_2 \cdot \ldots \cdot p_r. \tag{2}$$

Moreover, we can fix the order by requiring that $p_1 \leq p_2 \leq \ldots \leq p_r$.

Remarks. 1) The main ingredient of the proof of the Fundamental Theorem of Arithmetic is Euclid's Lemma.

2) From the theorem it is easy to see that the following two numbers are not equal, but it is much harder to determine which of the two is larger:

$$\underbrace{11 \cdot 13 \cdot 13 \cdot 17}_{31603} \neq \underbrace{2 \cdot 3 \cdot 23 \cdot 229}_{31602}$$