The Chinese Remainder Theorem

Theorem 5: Suppose that $m_1, m_2, \ldots, m_r$ are positive integers such that

$$\gcd(m_i, m_j) = 1, \quad \text{for all } i < j.$$

Put

$$m = m_1 m_2 \cdots m_r,$$

$$m_i' = \frac{m}{m_i}, \quad 1 \leq i \leq r,$$

and choose integers m_i^* such that

$$m_i' m_i^* \equiv 1 \pmod{m_i}, \quad 1 \leq i \leq r.$$

Then for integers $x, a_1, \ldots, a_r$, the simultaneous congruences

$$\begin{aligned} x &\equiv a_1 \pmod{m_1} \\ x &\equiv a_2 \pmod{m_2} \\ &\vdots \qquad\quad \vdots \\ x &\equiv a_r \pmod{m_r} \end{aligned}$$

hold if and only if

$$x \equiv a_1 m_1^* m_1' + \ldots + a_r m_r^* m_r' \pmod{m}.$$

Note: This method is explained in Master Sun Tzu's Arithmetical Manual (written between 273 - 473 A.D.) It is one of the earliest preserved Chinese textbooks on Arithmetic.