Solutions of $z^n = a$

Theorem 1: The equation

$$(1) \qquad z^n = a, \qquad \text{where } a \in \mathbb{C},$$

has precisely n distinct solutions $z_0, z_1, \ldots, z_{n-1} \in \mathbb{C}$. These all lie on the circle (centered at the origin) of radius $R = |a|^{\frac{1}{n}}$, and are given by the formula:

$$(2) \quad z_k = R(\cos\theta_k + i\sin\theta_k), \quad 0 \le k \le n-1,$$

where

$$\theta_k = \frac{\arg(a) + 2\pi k}{n}.$$

Recall: If $a = r(\cos\alpha + i\sin\alpha)$,

then $\alpha = \arg(a)$.

Note: Since the difference between successive arguments is constant, i.e.

$$\theta_{k+1} - \theta_k = \frac{2\pi}{n},$$

we see that the points $z_0, z_1, \ldots, z_{n-1}$ are spaced equidistantly on the circle.