Irreducible Polynomials

Definition: A polynomial $f(x) \in F[x]$ is called irreducible over F (or in $F[x]$) if $\deg(f) > 0$ and if its only factors are c and $cf(x)$, where $c \in F, c \neq 0$, is any non-zero constant. If $\deg(f) > 0$ and f is not irreducible, then f is called reducible over F.

Note: If $\deg(f) > 0$, then f is reducible over $F \Leftrightarrow$ there exists $g \in F[x]$ with $0 < \deg(g) < \deg(f)$ and $g|f$.

Theorem 7 a) Every linear polynomial $f(x) = x - a \in F[x]$ is irreducible in $F[x]$.

b) If f is irreducible in $F[x]$ and $\deg(f) \geq 2$, then $f(a) \neq 0$, for all $a \in F$.

c) If $\deg(f) = 2$ or 3, then f is irreducible in $F[x]$ if and only if $f(a) \neq 0$, for all $a \in F$.

Remark. The theorem shows that there is a partial relationship between the following two concepts:

(i) $f(x)$ is reducible in $F[x]$;

(ii) $f(x)$ has a root in F.

Indeed, we have (ii) $\Leftrightarrow$ (i) if $\deg(f) = 2$ or 3, but in general we only have (ii) $\Rightarrow$ (i).