The Principle of Induction

Building Block Law: The number $1 \in \mathbb{N}$ is the basic building block for all positive integers. In other words, every positive integer $n \in \mathbb{N}$ is a sum of 1's:

$$n = \underbrace{1 + \ldots + 1}_{n}.$$

This property leads to the following principle.

Principle of Mathematical Induction: Suppose that $P(n)$ is a statement depending on an integer $n \geq 1$. If:

(i) $P(1)$ is a true statement, and

(ii) whenever $P(k)$ is true for $k \in \mathbb{N}$, then $P(k+1)$ is also true,

then $P(n)$ is true for every positive integer $n \in \mathbb{N}$.

Example: Prove that $2 \mid (n^2 - n)$, for every $n > 0$.

Proof (by induction): Let $P(n)$ be the statement: $2 \mid (n^2 - n)$.

Step 1: $P(1)$ is true because $2|0 = 1^2 - 1$.

Step 2: Assume the $P(k)$ is true, i.e., that $2|(k^2-k)$. Then by D3, it follows that $2|((k^2 - k) + 2k) = (k+1)^2 - (k+1)$, so $P(k+1)$ is true.

Conclusion: $P(n)$ is true for all $n \geq 1$.